PRACTICALL GEOMETRY,

In Two Parts:

The first, shewing how to perform the foure Species of *Arithmeticke* (viz: ADDITION, SUBSTRACTION, MULTIPLICATION, and DIVISION) Together with REDUCTION, and the *Rule of* PROPORTION in *FIGURES*.

The second, containing A HUNDRED GEOMETRICALL QUESTIONS, with their SOLUTIONS & DEMONSTRATIONS, some of them being performed *Arithmetically*, and others *Geometrically*, yet all without the help of *Algebra*.

A worke very necessary for all Men, but principally for Surveyors *of* Land, Engineers, Military Architects, *and all other* Students *in the* Mathematicks.

By Captaine THOMAS RUDD, chiefe Engineer to *His late Majestie*.

Imprinted at *London* by J.G. for ROBERT BOYDELL, and are to be sold at his Shop in the Bulwarke neer the Tower, 1650.

To the Reader.

GEOMETRY, which *Philo* calleth, *the Princesse and Mother of all Sciences*, hath properly its name from *measuring the Earth*; and is called of the *Greekes*, Γεωμετρία, of γῆ, *i.e. Terra*, and μετρέω, *i.e. Metior*; and therefore was *Architas* an eminent *Geometrician*, called by *Horace*, *Mensorem*.

Et Maris, & Terræ, numeróque carentis arenæ.

This *Science* had its first originall in *Ægypt*, for when *Nilus* with his inundation drowned and confounded the limits of their *Lands*, certaine of the most ingenious Inhabitants were constrained

to find out the rules of *Geometry*, by the benefit whereof, (after the fall of the water) every man had his portion of *ground* reſtored to him againe. Out of *Ægypt*, *Thales* brought it into *Greece*, where it received great perfection, and almoſt extended to its preſent maturity.

The utility and benefit of this moſt admirable *Art* is of ſuch importance, that without the aſsiſtance thereof we can ſcarcely *live*, much leſſe *preſerve* our ſelves from danger, or hinder the *irruption* of a forreigne *Enemy*; for, (to omit the attaining of the *diſtances*, *magnitudes*, and *motions* of the *Cœleſtiall Orbes*, the probability of *the heroick Hypotheſis of Copernicus concerning the Earths motion*, & ſolving the *various* conjecturs of the *Ancients*, concerning its *ſpecies*; all wch have their evident proofe and demonſtration from *undeniable Geometry*) by help hereof the *Geodetian* may *meaſure*, *plot*, and

give the true *ſymmetry*, *ſcituation*, and *content* of your *Land*; ſearch out all kinde of *Longitudes*, *Latitudes*, and *Altitudes*, whether acceſsible or inacceſsible. The *Military Architect* may *fabricate* his *Fort*: The *Generall* or *Major embattaile* his *Army*: The *Engineer* plant his *Cannon*, convey his *Myne*, and *preſent* his *execution* to the diſadvantage of his *Enemy*: The *compleat Shadowiſt* can teach the *Sun* to trace out his *way* upon the *Earth*, and by the *ſhadow* of an *Axis* to point out unto us thoſe *Atomes* of time into which our *artificiall Day* is *artificially* divided.

I will not therefore goe about to apologize for this *Worke*, the Subject thereof being GEOMETRY; for, it ſhall ſpeak for it ſelf louder than either others friendſhip, or envy, the *name* and *employment* of the deceaſed *Authour* (if at leaſt it be ſafe to ſay thoſe men are *dead*,

who ever *ſurvive* in their *living Monuments*) being *garland* ſufficient to proclaime the *worth* within, in which you ſhall finde the moſt choiceſt *Problemes*, and niceſt *Queſtions* of the *Antient* and *Moderne Geometricians*, of Forreigne Nations, *grafted* upon an *Engliſh ſtocke*; the *Author*, like the painfull *Bee*, ſucking of the ſweeteſt *flowers* in the rareſt *Gardens*, and making *honey* thereof for his *Countries uſe*, which (had the *Almighty* ſpared his *life*) would have appeared in a more comely *dreſſe*.

The whole *Worke* is divided into *Two Parts*; in the *Firſt* whereof is ſhewed how to performe the 4 *ſpecies* of *Arithmeticke*, (viz: *Addition*, *Subſtraction*, *Multiplication*, and *Diviſion*, together with *Reduction* and *the rule of Proportion*) in *Figures*, by help whereof the *Surveyor* may finde many compendious *rules* for the *reducing*, *dividing*, and *adding* of his

plots, for the better expediting of his *Arithmeticall worke* in finding his *Contents*.

In the *ſecond Part*, is A HUNDRED QUESTIONS with their *Solutions* & *Demonſtrations*, ſome of them being *Arithmeticall*, and others *Geometrically* anſwered, yet all without the help of *Algebra*; the worth of theſe *Queſtions* will be beſt valued by thoſe, who after much *wandring* have at length ſate down *wearied* with the obſcure and toilſome *Labyrinth* of others. Let the *preſent* and *future* times enjoy the *benefit* of ſo *profitable* and *pleaſing* a *worke*, and at once *honour* the *Author*, and *thanke* the *Editor*.

PRACTI-

PRACTICALL GEOMETRY.

Probleme I.

To Adde or Joyne together Geometricall Figures.

THe adding of Geometricall right line Figures, doth consist chiefly in foure things:

Firſt, in Triangles, and Parallelograms of equall height.

Secondly, in many, and unequall angled, and ſpered Figures.

Thirdly, in like formed Figures.

And laſtly, in a given, and preſcribed manner, or a given forme.

Of Triangles.

THe addition of Triangles of equall height, is nothing elſe, but to adde their Baſes, or to make a Triangle of the ſame height, whoſe Baſe is as long as all the Triangles, that ſhould be added together. The reaſon is, That all Triangles, which have equall height, and equall Baſes, are equall, and that by

adding their Baſes together, are all included in one Triangle.

In like manner alſo, all Parellegrams of one height, by adding of their Baſes, are added into one Parallelogram.

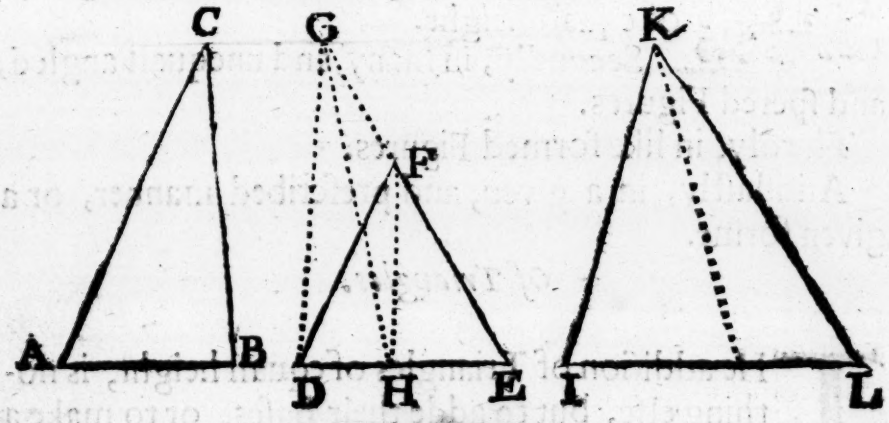
But concerning equiangled, or unequiangled, many ſided figures, they muſt firſt (by the foreſaid doctrine) be altered into Triangles, or Parallelograms of equall height, and then adde them as before is ſaid.

And to adde like formed Figures together, is underſtood by the 31 Propoſition of the 6 Booke of *Euclid*, and needeth no farther inſtruction.

And laſtly, to adde Figures according to a given or preſcribed manner: And firſt of the Adding of Triangles, as followeth:

Problеme II.

To Adde two Triangles of unequall height together, and to give their ſumme, in a Triangle.



Conſtruction.

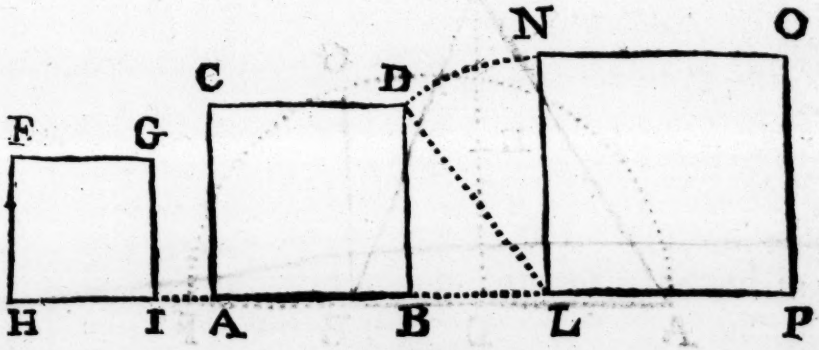
THe two Triangles given, are A B C and D E F, It is required to adde theſe two, and to ſhew their ſumme in one Triangle equall in height to A B C.

We

We have shewed in the 20 Probleme, how to change the Triangle D E F. into an equall height with the Triangle A B C, retaining still the same quantity of the triangle D E F, for which we have the triangle H G E. we have then in this addition nothing to doe, but to adde the base H E, of the triangle H G E, to the base A B, of the triangle A B C, and there comes, for a summe, the base I L, in the triangle I K L. Then I say, that the triangle I K L, is equall to the two unequall triangles A B C and D E F, and in height equall to A B C, which was required to be done.

Probleme III.

Two Squares being given to be added, to finde their summe.



Construction.

LEt there be to be added two Squares A B C D, and F G H I, and let one of the sides of the one square, as of A B, be prolonged at pleasure, and let the side of the other square be set upon the prolonged line, beginning from the point of the line prolonged, till the end of the square: As for example, we prolong the side A B infinitely, towards K, then we set the side F G, from the point B to L, and we draw the line D L. I say then, that D L is as the side of a square containing as much,

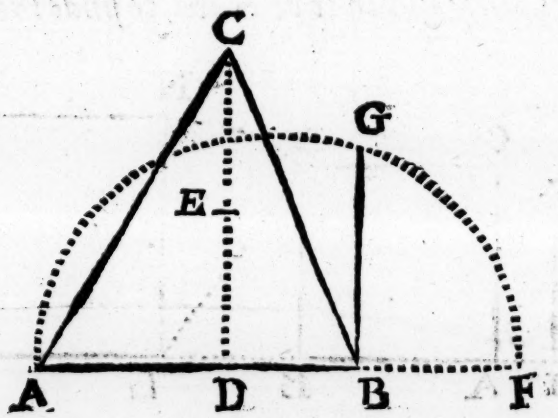
as the two ſquares given, A B C D and F G H I, together.

Demonſtration.

The Demonſtration is manifeſt, by the 47 of the 1 *Euclid*: For the ſquare which is made of the ſide L D, is equall to the ſquares made of the ſides L B, B D. But L B is the ſide of the ſquare F G H I, as alſo the ſide B D equall to the ſquare D A, wherefore the ſquare that is made of the line L D, is equall to the ſquares aforeſaid, which was required to be demonſtrated.

PROBLEME IV.

To change or reduce a Triangle into a Square.

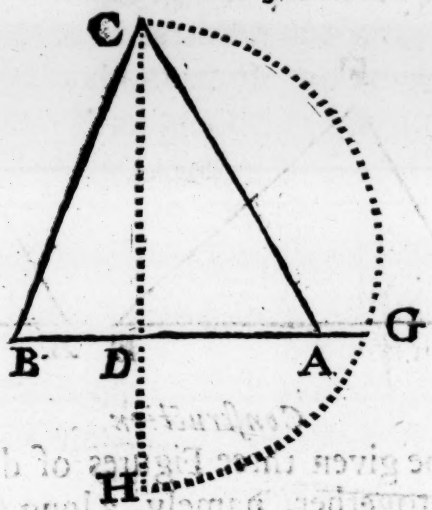


Conſtruction.

LEt the triangle be A B C, and let his perpendicular line C D, be divided into two equall parts in the point E, and ſet one of thoſe parts from B to F, then upon the whole line A F, make a halfe circle A G F, then from the point B raiſe a meane proportionall line, as B G, then I ſay that G B is the ſide of a ſquare, which containeth as much as the given triangle, by the 14 of the 2 book of *Euclid*.

Another

Another way.



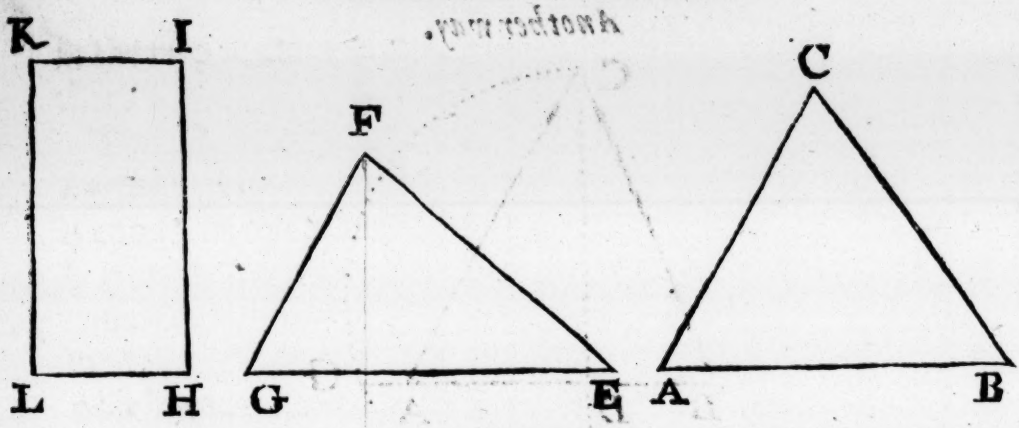
PRolong the perpendicular C D, and let the half of A B be ſet from D to H, and upon C H deſcribe a half circle C G H, and let D B be alſo prolonged to G, then is D G alſo the ſide of a ſquare, equall to the ſide triangle A B C.

Note,

That how to make a ſquare upon a given line for the ſide of the ſame, is ſhewed in the 46 Propoſition of the 1 Booke, and how to adde two ſquares is ſhewed in the 15 of the 2 Booke of *Euclid*: therefore wee omit to repeate the ſame in this place.

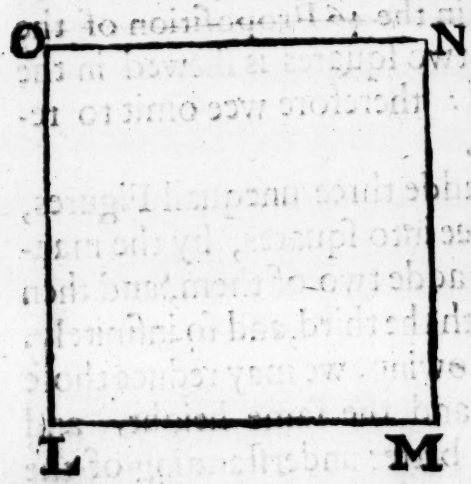
But if it were required to adde three unequall Figures, we may reduce them all three into ſquares, by the manner before ſhewed, and then adde two of them, and then in the ſame manner, doe with the third, and ſo infinitely.

Or by this ſecond way following, we may reduce thoſe unequall Figures into one and the ſame height, and then, as before: and for the better underſtanding of the ſame, here doth follow yet one Example of three Figures unequall, both in forme and quantity, to be added together, into one Figure.



Construction.

LEt there be given three Figures of divers kinds, to be added together, namely, a long ſquare, a Scalenum, and an Equilaterall triangle, and let the long ſquare be L H I K, and the Scalenum G E F, and the Equilaterall let be A B C. To adde all theſe together, firſt reduce the Scalenum into a ſquare, by the laſt Probleme going before, as alſo the Equilaterall; then adde thoſe ſquares; and laſtly reduce the long ſquare, into a ſquare, by the 23 Probleme; and then adde that ſquare to the former,

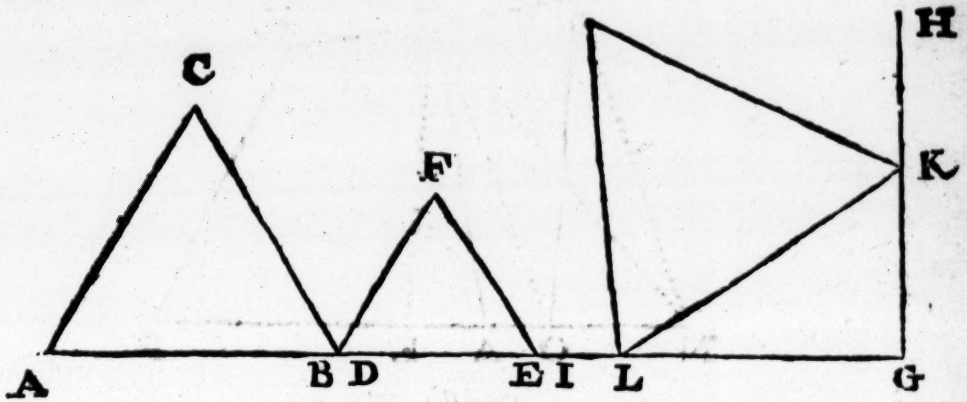


& there ſhall come for the ſume, the ſquare L M N O, you may alſo reduce the two triangles A B C and G E F given, firſt into a triangle of the ſame hight, by the 20 Probleme, and then into a long ſquare by the 21 Probleme, and then adde thoſe long ſquares together, by adding of their baſes; and laſtly, reducing that totall long ſquare, into a Geometricall ſquare as was required.

But

But Note, that thoſe Figures which are proportionall, may be added together without reducing into ſquares; as ſhall be ſhewed in the example following:

Two equilaterall Triangles being given, that is A B C and D E F, to adde the ſame.

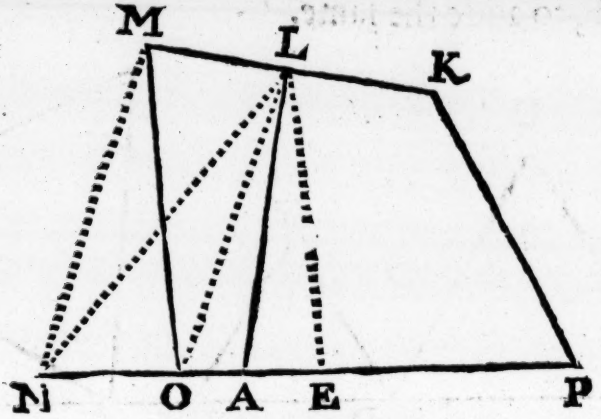


Conſtruction.

LEt the two Equilaterall triangles be ABC and DEF, which we would adde together, and of the ſumme we would make a triangle like to the given, and conteining as much as the two ſaid triangles: To doe the ſame, draw a baſe line, as G I, and at the end thereof (by the 2 Probleme) draw a line perpendicular, as G H, let the baſe A B be ſet from G toward I, comes to L, then alſo the baſe D E be ſet from G toward H comes to K, then draw K L, which is the ſide of an equilater triangle conteyning as much as the two triangles given: as appeareth by the 31 of the 6 of *Euclide*.

PROBLEME V.

To adde divers Figures together, according to a prescribed manner.



Construction.

LEt there be this Figure A L K P given, to be added to any given Figure, as to this Figure LAOM and let it be required that the sides K L and P A be prolonged, and that the said Figure be added to the side A L, and come from the point A to the point O, on the prolonged base P A, and on the head from L, and so forth; with the prolonged line K L right out.

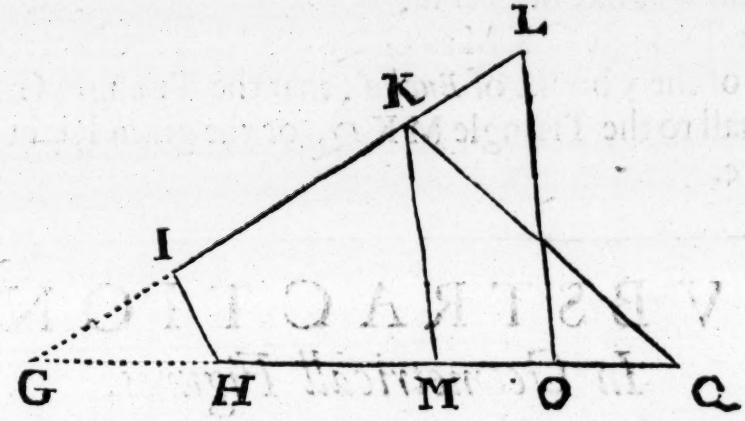
Let the Figures to be added, be reduced (by the former instruction) into a triangle of the height of E L, and set in a right line with P A as the triangle A N L, then from O draw a line to L, and from N draw another line parallel to O L as N M, cutting the prolonged line K L in the point M, and thence draw a line to O. Then I say, that the Figure O M K P is equall to the Figures A L K P together with the given Figure (which was changed into the triangle) N L A, which was required.

Demonstration.

The given Figure is equall to the triangle A N L, namely,

namely, O L A & O L N together, and O L N, O L M are equall, (by the 37 of the 1 Booke of *Euclid*) It followeth that O M L A is equall to the given Figure.

Another.



THere is a Figure given, as HIKM. To the which it is requir'd to adde a right line Figure, parallel with the side M K, and that the two sides I K, HM. be prolonged right out; to doe this, produce the sides K I and M H till they concurre together in G, then change the Right line Figure into a Triangle of the same height which the Triangle G K M is of, which in this, is the Triangle M K Q, their Bases being joyned in one Right line, as G Q. Then by the 13 of the 6 of *Euclid*, finde a meane proportionall line between G Q and G M, and set that from G towards Q comes to O, and from O draw a Right line from thence parallel to M K, and cutting the produced line I K in L, then is the given Figure added to the Figure H I K M, like as was required.

Demonſtration.

For as much as there are three lines proportionall, as G Q, G O, G M, it is evident (by the 19 of the 6 of *Euclid*) that the Triangle G L O hath proportion to the Triangle G K M, as the line G Q to G M: and by the 1 of the 6 booke of *Euclid*, is the Triangle G K Q alſo to the Triangle G K M as G Q to G M. It followeth by the 9 of the 5 booke of *Euclid*, that the Triangle G L O is equall to the Triangle M K Q, or the given Right line Figure.

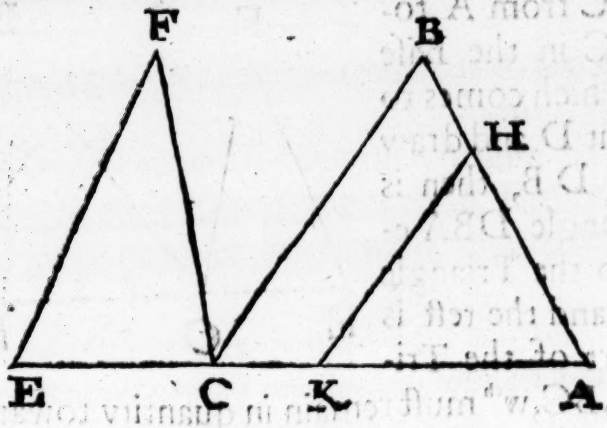
SVBSTRACTION,
In Geometricall Figures.

The eighth Diſtinction.

THe ſubſtracting of Geometrical Figures, conſiſts alſo in foure grounds (like as Addition) is already ſhewed; and may alſo by the Inſtructions of the ſame, be ſufficiently underſtood, in as much as the one, is the right contrary to the other; For like as there, by the adding of the Baſes of Triangles, and Parallelograms, of equall height, the worke is finiſhed; even ſo here muſt be underſtood, that the Figure from whence the Figure given, ſhall be ſubſtracted, muſt be greater, than that which muſt be ſubſtracted: wherefore we will here ſet divers examples, as followeth.

Probleme VI.

To substract one Triangle from another.



Construction.

LEt the Triangle given be A B C, from which is to be substracted, a Right line Figure, (which Right line Figure by the former doctrine, is changed into a Triangle, in height equall to the same Triangle given, A B C, which Triangle, is C D E) from the Angle C, so that the line of Division comes to be parallel to the side C B. To doe the same,

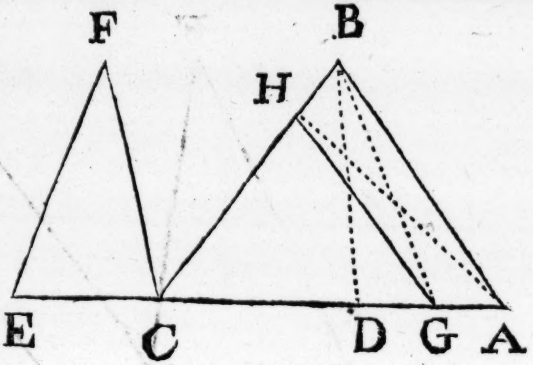
Seeke a meane proportionall between the Bases of the two Triangles A C and C E, and set the same from A towards C comes to K, and from the point K draw a line to B C, cutting the side A B in the point H, then doth the same K H cut off from the Triangle A B C a Triangle A H K equall to the Triangle C E F, which was required.

The Demonstration is evident by the last example, in Addition, before.

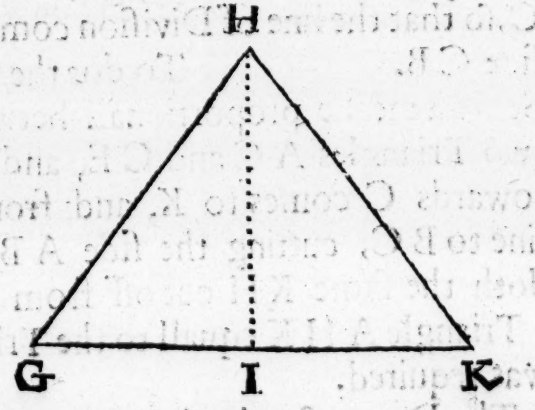
But if it be required, that the aforesaid Triangle (now

reduced into equall height) ſhall be cut off, not from any Angle, but to be laid out parallel to the one ſide, as here in this Figure, from the ſide A B. To doe this,

Set the length of the Baſe E C from A towards C in the Baſe A C, which comes to the point D, and draw the line D B, then is the Triangle DBA equall to the Triangle E F C, and the reſt is that part of the Triangle ABC, wch muſt remain in quantity towards C. But the line of Diviſion B D muſt be brought parallel with the line A B, (by the 20 Probleme, and the 45 of the 1 of *Euclid*) as here alſo appeareth by the line G H, and ſo is the Triangle CEF cut off from the Triangle ABC with the line GH parallel to the ſide AB, as was required.



But if it were ſo, that in the ſubſtracting, it were required, that the line of Diviſion ſhal come to ſtand perpendicular upon the Baſe of the triangle, that is, or parallel with the perpendicular of the triangle: As for example, in this Triangle GHK, the lines to be parallel to the perpendicular I H, in this there is no difference in working, to that before ſhewed, for if the Triangle



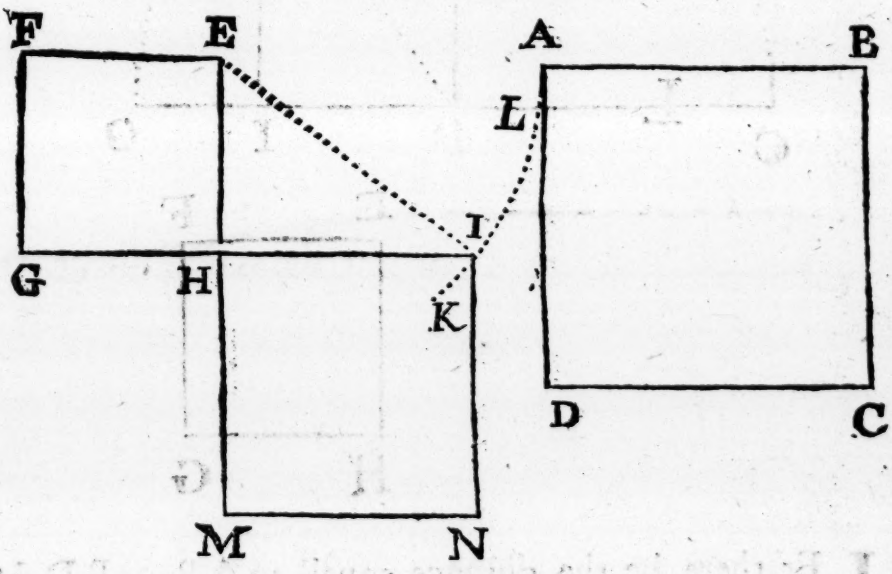
which

which is to be ſubſtracted, his Baſe be ſhorter, then the part of the Triangle from the point (on that ſide where the ſame is to be ſubſtracted) to the perpendicular of the Triangle I H, then is the worke all one and the ſame, with the firſt manner, as here above in the firſt Figure is ſhewed.

But if the Baſe of the Triangle which is to be ſubſtracted be longer than that part, ſo that you muſt goe paſt the pendicular I H towards the other Angle, then is the working all one with that of the ſecond Figure, which every one that intendeth to practice may examine by himſelfe.

Probleme VII.

Two like Figures being given, to ſubſtract from the greater a Figure equall to the leſſer.



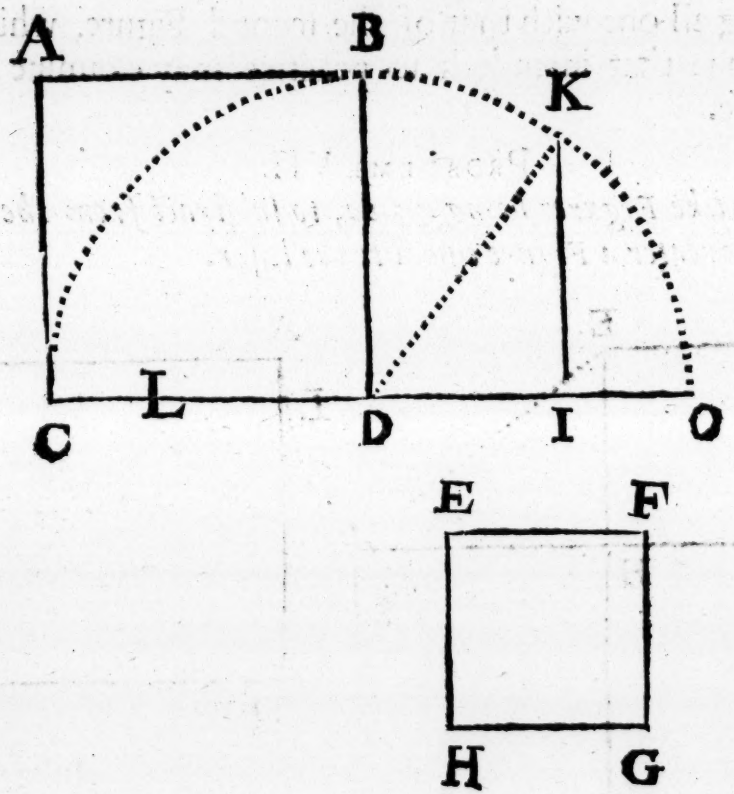
Conſtruction.

LEt there be two Squares, the greateſt A B C D, and the leſſer E F G H, let the ſide G H be prolonged towards I, and from the point E at the di-

ſtance A B, draw the arch line K L I cutting the prolonged line in the point K, then I ſay that the line H I is the ſide of a ſquare, the reſt of this ſubſtraction.

The Demonſtration is manifeſt, for the Square E H and H I together are the ſquare E I, by the 47 of the 1 of *Euclid*, wherefore the Square of E I is equall to E H and H I, which was required to be done.

Another way.



LEt there be the diſtance equall to A B, or B D ſet from the point D in O, and deſcribe a halfe circle C B O, and let there be ſet the ſide of the ſquare E F G H from D toward O in the point I, and let there be ſet the meane proportionall between C I and H G or I O, and raiſe

raiſe a perpendicular from the point I, as IK, which is the ſide of a Square, the reſt of this ſubſtraction.

The reaſon of this worke is manifeſt, for D K is equall to the ſide of the Square A B C D, and the ſame D K is equall to the Squares which are made of the ſame lines D I and I K, (by the 47 of the 1 of *Euclid:*) But D I is equall to the ſide of the Square E F G H, (by *Hypotheſe:*) K I is therefore the difference of the Squares A B C D and E F G H.

Note,

We could here give as many examples in Subſtraction, as we have done, in Addition here before, that is, how to ſubſtract one Triangle from another, or a Square from a Triangle, and the like things: But they which doe well underſtand the ſame in Addition, ſhall eaſily underſtand the ſame alſo in Subſtraction: wherefore, we for the preſent omit the ſame.

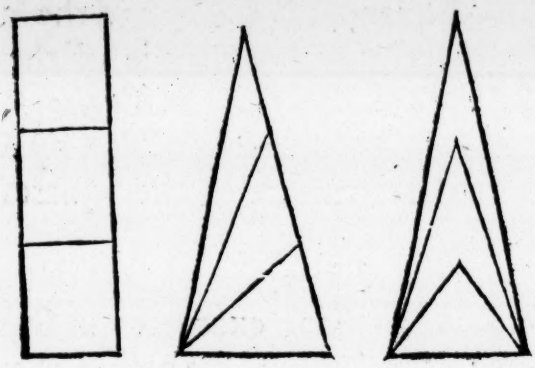
MULTIPLICATION,
In Geometricall Figures, by three waies.

The Ninth Diſtinction.

ALL Right line Figures may be increaſed generally by three manner of wayes: Firſt, by the former inſtructions, to alter the ſame into a Triangle or Parallelogram, and then the Baſes thereof ſo many times increaſed, as you would have the Figure increaſed; and then upon the increaſed Baſe draw the Triangle or Parallelogram, equall in height with the firſt Triangle or Parallelogram, and that Triangle or Parallelogram

ſhall

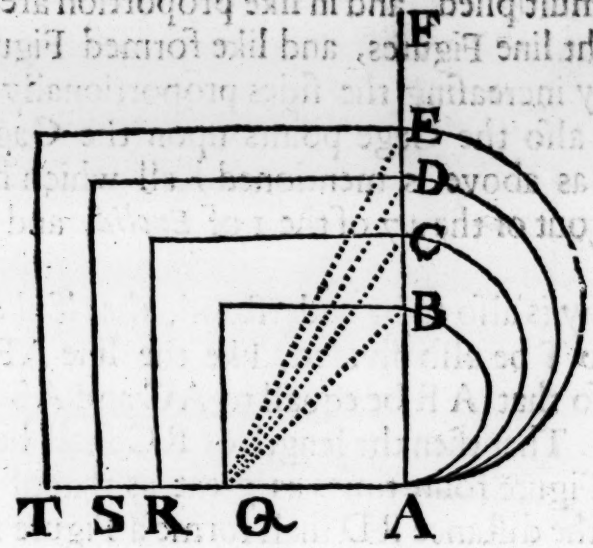
ſhall be ſo many times greater, as the Baſes was increaſed of the ſame. This working is evident, as appeareth by



the 38 Propoſition of the 1 Booke *Eutlid*; and the 1 Propoſition of the 6 of *Euclid*, (likewiſe it is hereby given to underſtand, that the Triangles and Parallelograms ſtill remaining upon the ſame Baſes, by increaſing of their height, are thereby alſo multiplied) like as in theſe Figures plainly appeare, where one of the Perpendiculars or up-going ſides, being three times increaſed, whereby the Figures are three times as great as before; and afterward of the ſame Triangles & Parallelograms may be made againe a Right line Figure, which ſhall be like formed to the Figure given, or to any other forme required.

The ſecond manner is, to make a Right angled Triangle, whoſe two ſides which doth include the Right angle, each in length equall to the one ſide of the Figure, which is required to be multiplied, and then upon the Hypothenuſa, that is, upon the ſubtending ſide make a Figure in forme, like to the Figure given (by the 31 of the 6 of *Euclid*) by which alſo can be underſtood, how to increaſe any Figure, as many times greater, as is required.

But



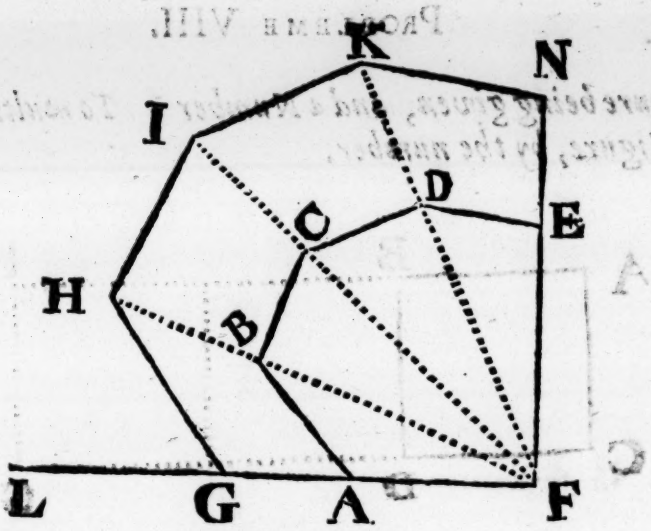
But to ſhew the ſame more plainly by Figure, then draw at your pleaſure a Right line, as in this, the line A T, and from A draw another line Right angled upon A T as A E, then draw from A towards F, and towards T, the length of a ſquare or ſide of another Figure, which is required to be increaſed in like forme, which for Example, comes in the line A T from the point A to Q, and in the line A F it comes to B, and draw the line Q B, which length Q B ſet from A towards F upon the line AF comes to C, then is the line AC the ſide of a ſquare (or other like formed Figure) twice as great as the length of AB, alſo the length Q C taken from A comes to D, then is A D the length of the ſide of a Figure three times as great as the Figure made of A B, and ſo forth; for the line Q D ſhall be the ſide of a Figure foure times as great as that whoſe ſide is AB, which length QD doth extend in the line AF from A to E, ſo that AE twice as long is as AB, in ſo much as you may ſee on the one ſide of this Figure, the Squares, and on the other ſide, the

Circles, multiplied, and in like proportion are the ſides of all Right line Figures, and like formed Figures multiplied, by increaſing the ſides proportionally as above is ſaid, as alſo the Gage points upon the Gage Roode ſet out, (as above is mentioned) all which hath their grounds, out of the 47 of the 1 of *Euclid*, and 31 of the 6 booke.

Whereby is alſo to be underſtood, that if the line AT from A to T be alſo divided, like the line AF is from A to F, ſo that A R be equall to AC and AS equall to AD, &c. That then the length of RC ſhall be alſo the ſide of a Figure foure times as great as that of AB or of AQ, and the diſtance RD ſhall forme a Figure five times as great as AB by reaſon that AR two and AC two doth produce together four, and alſo A R two, and AD three together makes five, and ſo of the reſt.

The third manner to multiply is thus, make a Right line, which ſhall be as many times the length of one ſide of the Figure (which you would multiply) as you deſire to have the Figure greater, and then betweene that whole line and the ſide of the Figure given, ſeek a meane proportionall, by the 13 of the 6 booke, and upon the ſame meane proportionall line, draw a Figure, like formed to the Figure given, by the 18 of the ſaid 6 booke of *Euclid*, the ſame Figure ſhall be ſo many times greater than the given Figure, as the ſide was lengthned, which by this Example may be better underſtood.

This



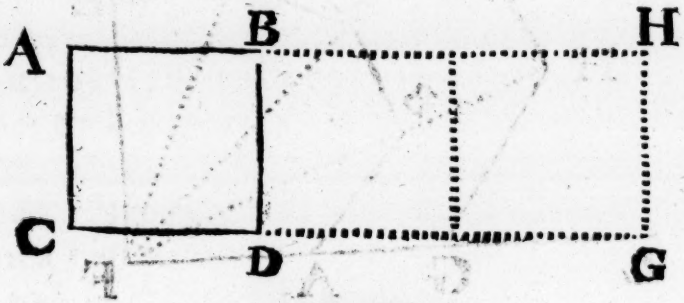
This fix angle ABCDEF is required to be increaſed three times greater than the ſame now is, which to doe, extend the one ſide three times as long, (which let be the ſide AF) comes to L, ſo that FL be three times as long as FA, then the meane proportionall between the whole line and the line FA, as the line FG is the length of the ſide required, then upon the ſame line FG deſcribe a Figure like formed to ABCDEF, which Figure is GHIKNF, the ſame is three times as great, or triple to the aforeſaid Six angle given ABCDEF.

Demonſtration.

For as much as the lines FL, FG, FA, are proportionall (by the 19 and 20 Propoſitions of the 6 booke) the proportion of the figure GHIKNF to the figure ABCDEF is equall to the proportion of the line LF, to the line FA, but the line F L is three times FA, wherefore the figure GHIKNF is alſo three times the figure ABCDEF.

Probleme VIII.

A Figure being given, and a Number: To multiply the Figure, by the number.

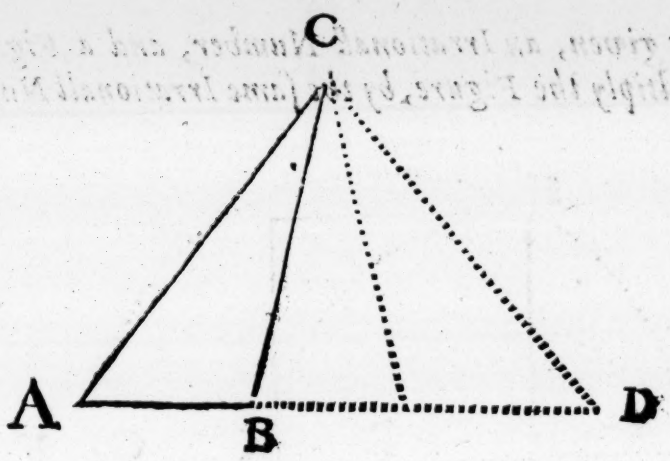


Conſtruction.

LEt the Figure given be ABCD, which we would multiply by three, that is, would make the ſame three times as great as the ſame is, let the lines AB be prolonged to H, and CD to G, and draw the line HG at the diſtance of AB taken three times, which comes to H, then is the Figure AH, GC, the product Figure, required, ſuch proportion as the line AH hath to AB or CG to CD, ſo is the proportion of the figure AG to the figure AD, but the line AH is to the line AB triple, by conſtruction, wherefore the ſuperficies AG is to the ſuperfices AD alſo triple.

The

The ſame alſo in Triangles.

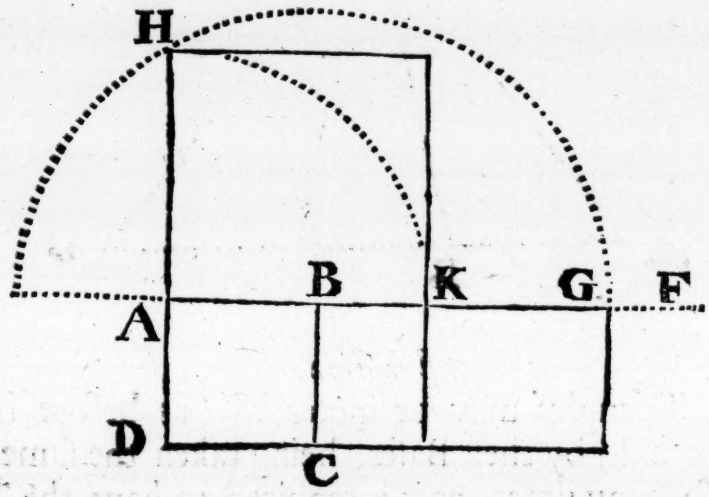


Conſtruction.

ALL Triangles may be increaſed (as before hath been ſaid) by their Baſes, being taken the ſame diſtance, ſo many times, as it is required to have the Triangle made greater, and then upon the increaſed Baſe and of the former height given, to make another Triangle. Example, the Triangle given is ABC, the ſpace AB is ſet three times from A to D, and upon the Baſe AD is made the Triangle ADC upon the ſame height as before ABC, then I ſay that the Triangle ADC is three times as great as the Triangle ABC: the thing is manifeſt and needeth no further demonſtration.

Probleme IX.

Being given, an Irrationall Number, and a Figure: To multiply the Figure, by the ſame Irrationall Number.



Conſtruction.

LEt the Figure propoſed be ABCD, and the number, let be √ three.

Let the ſide of the figure AB be prolonged forward toward F, and let the ſame ſide AB be ſet three times from A comes to G, then between AG and AD finde a meane proportionall, which let be AH, upon which meane proportionall forme a Figure, like the Figure propounded, the ſame is the product Figure required, as KD is √ three times greater than BD.

DIVISION,
In Geometricall Figures.

The Tenth Distinction.

THe dividing of Geometricall Figures may chiefly be required after foure ſundry manners of working, whereby all Triangles and Parallelograms may by dividing their Baſes, for in Parallelograms if we divide the Baſe and upper ſide or oppoſite ſide, and draw lines parallel: And Triangles from the upper Angle to the parts of the Baſe.

Secondly, all Right line Figures from a point given.

And thirdly, with parallel or ſtanding lines, that is, with lines ſtanding all one way, or perpendicular.

And fourthly, according to a given proportion.

This worke doth chiefly conſiſt upon three grounds: The firſt is, (by the 1 Propoſition of the 6 book of *Euclid*, is proved) that all Triangles and Parallelograms are divided according to the proportion of their Baſes, whereby the proportion alſo of any two Figures may be knowne if each of them be brought into a Triangle of equall height, and thereby to divide any Figure according to the ſame proportion, by the 10 of the 6 book of *Euclid*.

Secondly, by the 37 and 38 Propoſitions of the 1 booke, is proved, that all Triangles which have equall height, and ſtand upon equall Baſes are equall, by which meanes the line of Diviſion may be laid and altered to any point required.

Thirdly, (by the 19 and 20 of the 6 booke) it is evi-

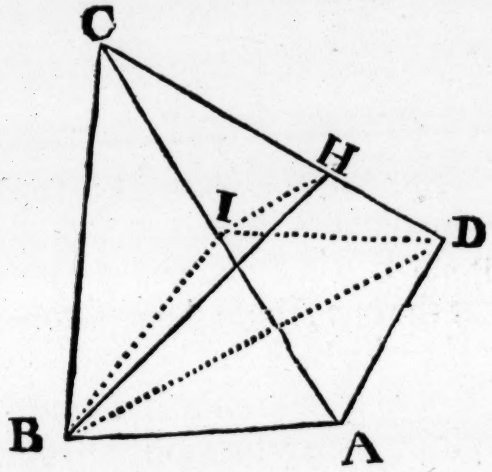
dent,

dent, that if three lines be proportioned, then the like formed Figures made of the firſt line, is in proportion to that made of the ſecond line, like as the proportion of the firſt line is to the third line, whereby one Triangle which hath equall height with another, that the ſame, and alſo all Right line Figures can be made like formed, as may be underſtood by that which followeth.

To declare this matter more at large: It is firſt to be underſtood, that if the Baſe of any Triangle be divided into any parts, equall or unequall, howſover, and that the lines of Diviſion be drawne from thoſe points of Diviſion to the upper Angle, the ſame lines doe divide the Triangle into ſuch parts as the Baſe is divided into, whether equall or unequall, in like manner alſo, in ſuch parts as ſides of the Parallelograms are divided into, one to another, and lines drawne right from one to another, the Parallelograms are divided thereby into ſuch parts, which alſo hath place in foure angled Figures, where the two ſides upon which they fall, (that is) where the lines of Diviſion ſhall come, are parallels one to another, all which, by the 1 Propoſition of the 6 booke is evident, and needeth here no further declaring.

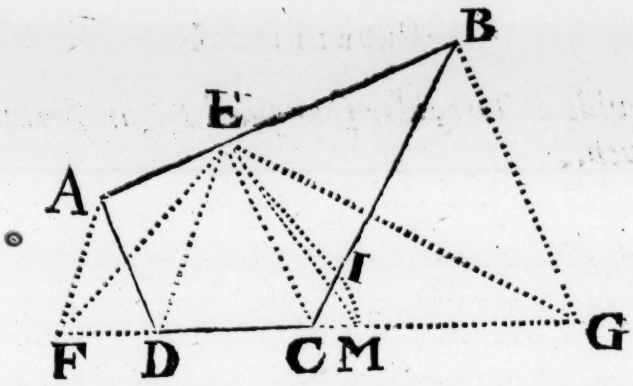
Probleme X.

To divide an Irregular four-sided Figure from a point given.



Construction.

LEt the Irregular four-sided and four-angled Figure be ABCD, and let the point given in the same be B, it is required from the point B to the side CD to divide the same into two equall parts; to doe the same, draw the line AC from the one Angle to the other, doe the like with BD, then divide the line AC into two equall parts in I, and from I draw a line parallel to BD cutting the side CD in the point H, as the line IH, and then from H draw a line to B: then I say that the line BH doth divide the same foure Angle into two equall parts, as was required.



BUt if it be required to divide the four Angle ABCD into two equall parts from a point in the one ſide, as from the point E, then change the ſame Figure into a Triangle, that the one Angle thereof come in the ſame point E, and that the Baſe D C on both ſides be produced forth right out, like as in this Figure; then by the ſecond example of the 44 Propoſition of the 1 booke of *Euclid*, make a Triangle equall to the ſame Figure whoſe height is the ſaid point E, as the Triangle F E G, then divide the Baſe FG into two equall parts, the midſt is the point C (in this Example:) then the Triangle FEG is equall to the foure Angle ABCD, then by the working of the ſaid 44 of the 1 booke, will the line EC divide the ſame into two equall parts.

But if it chance ſo that the line of Diviſion ſhall fall on the ſide BC like as it doth in the former Figure in the point I, then to find the ſame point I in the line BC draw from the midſt of FG (that is) from M to E a line, as ME, and from the point M draw a line parallel to CE cutting the line CB in the point I, and then from I to E draw the line of Diviſion as I E, then doth this line E I divide the foure Angle ABCD into two equall parts, that is to ſay, the Figure EICDA, and the Triangle EIB.

Demon-

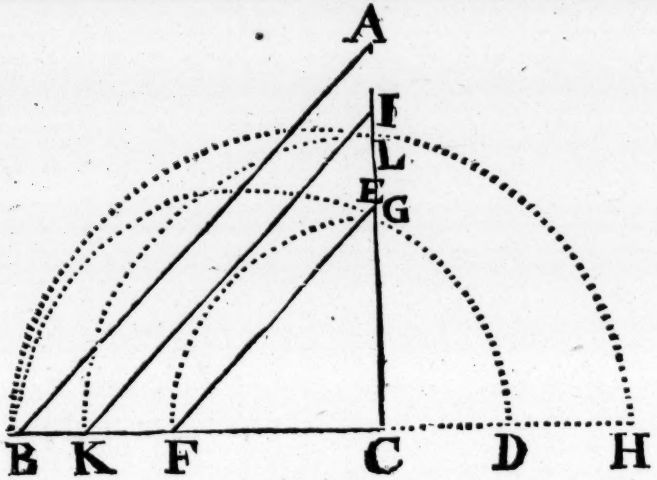
Demonstration.

First, let there be drawne the lines IB, ID, and for as much as by the 38 Proposition of the 1 of *Euclid*, the Triangles IBC, IBA, are equall, as also the Triangles IDC, IDA, then are the Figures IBCD and IBAD also equall; for as much then as the Triangles BID, BHD doe stand upon one Base & between two parallel lines, then are they equall, by the 37 of the 1 of *Euclide*, and therefore taken the one for the other, the Figures BHC and BHDA are also equall.

In the second Figure, by the parallel lines in the Figure can be understood that the Figure DAEM is the halfe of the foure Angle ABCD and the Triangle EIC within the Figure AEICF (by reason of the parallel lines CE, MI) is equall to the Triangle IMC, (which comes without the Figure with one Angle) wherefore the Division is true which was to be demonstrated.

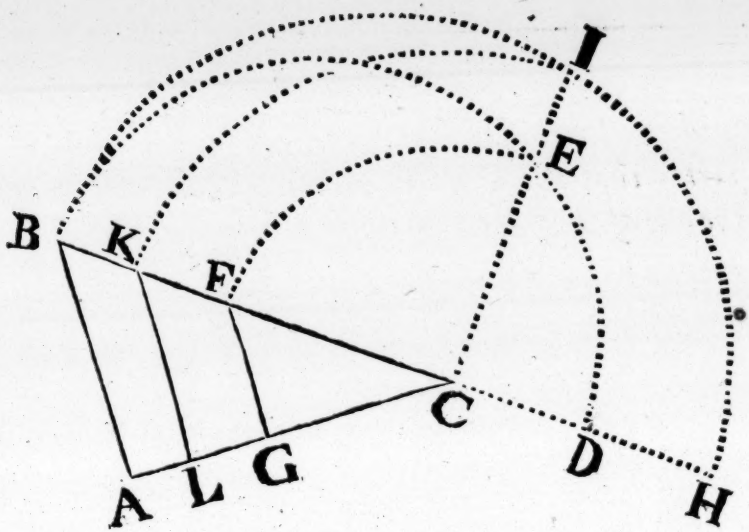
Problem XI.

A Triangle being given: to divide the ſame into three equall parts with lines parallel to one of the ſides.



Conſtruction.

LEt the Triangle given be ABC, it is required to divide the ſame lines parallel to the ſide AB, let the ſide A B or B C be divided into three equall parts (or into ſo many parts as is required to divide the Triangle into) and ſet two of thoſe parts upon the prolonged ſide B C, then between the whole line B C and the part C D ſeek the meane proportionall C E, which being ſet from C upon the line BC comes to F, and from this point F draw a line parallel to A B according as was required, and doe with the reſt of the Diviſions in like manner, how many ſoever: the Demonſtration is manifeſt, by the 15 of the 6 booke of *Euclid.*

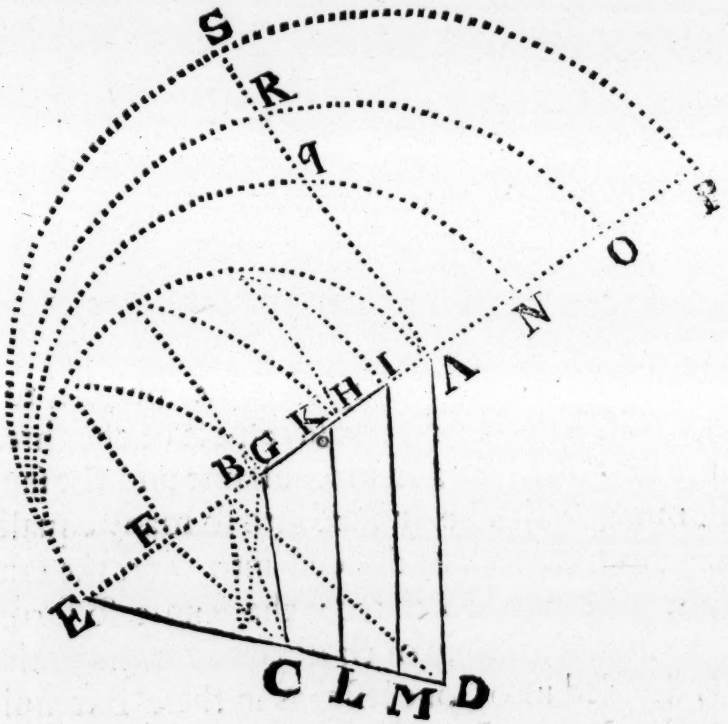


THe dividing of a Triangle by lines parallel to one of the ſides may be divers, in reſpect of the three ſides; but the working is in a manner all one and the ſame: for in all, muſt the ſide be divided into ſo many equall parts, as the Triangle ſhall be divided into, and the ſame ſide muſt be prolonged forth from the Angle ſo farre that it may containe ſo many of thoſe equall parts as the other line is divided into (one leſſe) as in theſe Examples, the line BC is divided into three equall parts, and the prolonged line conteineth two of them from C, that is, D and H, and then between the whole line and the one part ſeeke the meane proportionall for the firſt Diviſion line, and againe between the whole line, and the two equall parts ſeek the meane proportionall for the ſecond Diviſion line, and ſo forth to the laſt; thus farre the worke is

common, the difference is, in taking the ſides on which the parts ſhall be ſet, that the Diviſion lines may come to be parallel to the ſide required.

Probleme XII.

To divide an Irregular foure Angle in ſuch ſort into equall parts, that the lines of Diviſion ſhall come parallel to the one of the ſides.



Conſtruction.

LEt the ſides AB & CD of the quadrangle ABCD be prolonged to E, then ſee what proportion the part BCE hath to the whole quadrangle ABCD, which in thisexample let as one to three, that is, BCE one and ABCD three, theſe added together comes foure for

the

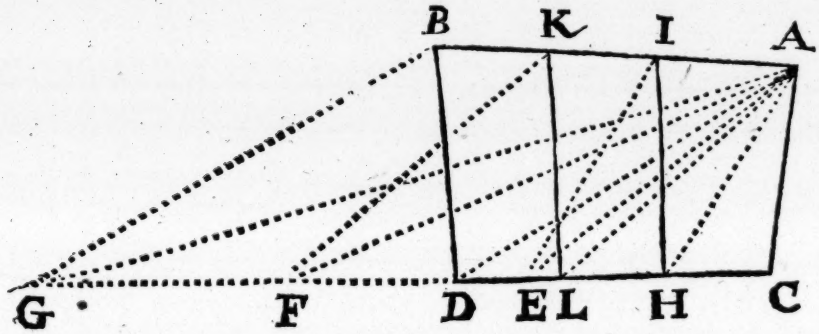
the whole Triangle A E D, wherefore we divide the whole line AE in foure equall parts, and then proceed as before in the Triangles, ſetting upon the prolonged ſide AP three of thoſe parts comes to N and O, and then betweene the whole line A E and the part A N ſeeke a meane proportionall, but for as much as A N doth fall without the quadrangle, you may omit the ſame, and ſeek the other parts.

And if it come ſo that the quadrangle doe containe more than three fourth parts of the whole Triangle AED, then we divide the Baſe AE in ſuch ſort that the *Area* of EBC with the one third part of the *Area* of the quadrangle be as the leſſer part to the greater: as for Example, let the Area of the quadrangle ABCD be twelve, then is one third part thereof foure; and let us ſuppoſe that the part EBC be two, we take the one third of twelve which is foure, and we adde the ſame to the two comes ſix for the one part, and the two third parts of twelve remaining makes eight, and we divide the line AE in ſuch proportion as ſix to eight, ſix in which is the Area of BEC and one quarter of twelve, and eight the two third parts of twelve the Area of the quadrangle ABCD, and then doe as before.

And for the ſecond Diviſion of the ſame do thus, adde eight to two comes ten, then we divide once againe the line A E in ſuch proportion as ten to foure, and ſo we have the three parts required, the Demonſtration is manifeſt by the 25 Propoſition of the 6 booke of *Euclid*.

Probleme XIII.

Being given an Irregular Quadrangle: to divide the ſame into three equall parts, in ſuch ſort that one of the ſides be alſo divided into three equall parts.

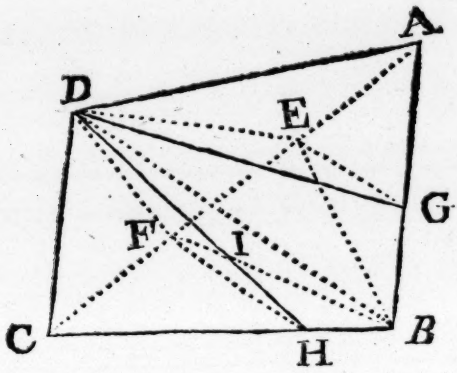


Conſtruction.

LEt the Irregular Quadrangle be ABCD, which being reduced into the Triangle ACG, which Quadrangle the ſide CD is divided into three equall parts in H and L, let alſo the whole Baſe CG of the Triangle A C G be divided into three equall parts in E and F, then let there be drawne the line AH, and from the point E draw a line parallel to the ſame AH as EI, then draw HI, then is HIAC the firſt third part; then draw L A, and from the point F draw a line parallel to L A as F K, then draw the line K L and ſo is the Quadrangle given divided into three equall parts, as was required to be done, by the 37 of the 1 of *Euclid*.

Probleme XIV.

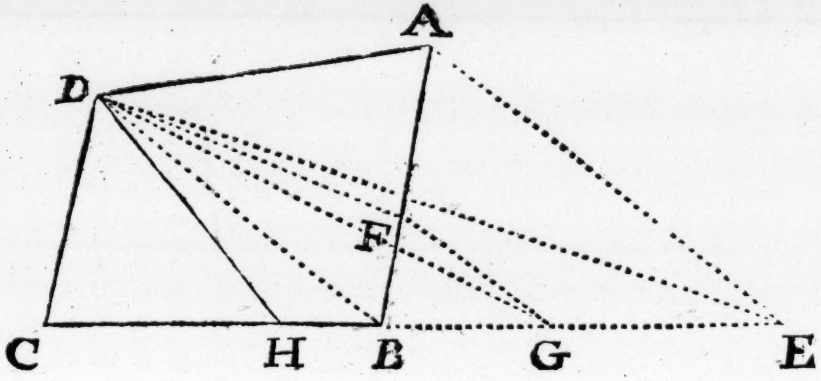
To divide an Irregular Quadrangle into three equall parts, by lines drawne from one Angle.



Conſtruction.

LEt the Quadrangle Irregular be ABCD, it is required to divide the ſame into three equall parts from the point D, let there be drawne the Diagonall lines A C and B D, and divide the Diagonall line A C into three equall parts in the points E and F, and from the ſaid points let lines be drawne parallel to the other Diagonall BD as the lines EG and FH, and alſo draw the lines DE and DF, then where the parallel lines EG and FH doth cut the ſides AB in the point G, and BC in the point H, draw from the Angle D right lines DG and DH, and you have the Diviſion required, that is to ſay, DAG the firſt part, DGH the ſecond, and DHC the third part.

Another way.

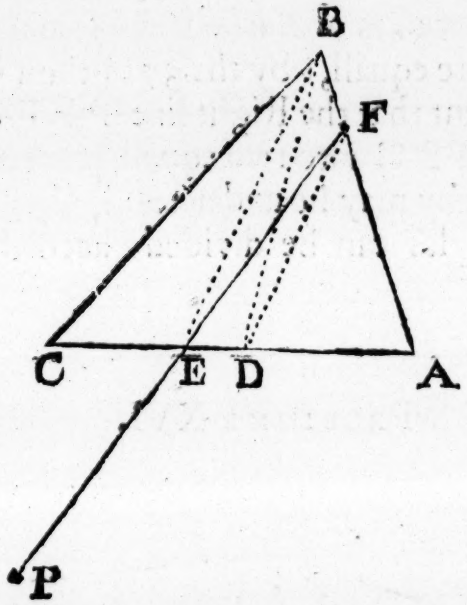


REduce the Quadrangle into a Triangle, as CED, divide the Base CE into three equall parts in H and G, and draw the lines D H, D G and D E, which doth divide the Triangle into three equall parts; then draw the line G F parallel to B D, and then draw D F, and so is the Quadrangle divided into three equall parts, that is, into D C H, H D F and F D A from the Angle D, as was required.

PROBLEME

Probleme XV.

To divide a Triangle into two equall parts, or unequall parts, with a Right line drawne from a point, taken without the ſame Triangle.



Conſtruction.

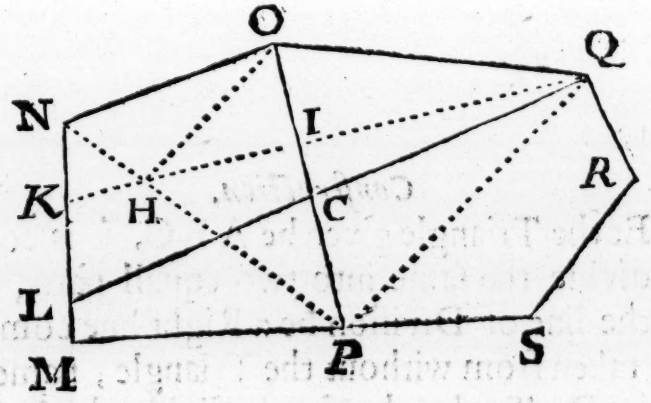
LEt the Triangle given be A B C, it is required to divide the ſame into two equall parts, and that the line of Diviſion be a Right line cõming from a point taken from without the Triangle; namely, from the point P: To doe the ſame, divide the Baſe A C in two equall parts in the point D, and from D a Right line drawne to the ſide A B in F, this point F removing this way, and that way, till you can from the ſame point F ſee the Right line from the point P, to cut the ſide of the Triangle A C in ſuch ſort, that from the ſame point

of cutting a line drawne from the ſame point of cutting in E to the Angle B, be parallel to the line D F, which being ſo done, then doth the Right line P E F divide the Triangle A B C in two equall parts F E A and F E B C.

Demonſtration.

For as much as the line B D doth divide the Triangle A B C into two equall parts (by the 38 of the 1 booke, and 1 of the 6 bookes of *Euclid)* and that the Triangles E B D, E F B, are equall, (by the 37 of the 1 book of *Euclid)* it is evident that the Right line P E F doth divide the Triangle A B C into two equall parts, F E A and E F B C, whereby may be underſtood, that in this manner the Triangles can be divided, according as is required.

Probleme XVI.



THere are two parcels of Land lying one by another, as theſe parts M N O P, and P O Q R S, the Owners of theſe two parts of Land doe agree that the ſame Land ſhall be altered in ſuch ſort, changed

changed one with another, that firſt thoſe two parts ſhall be put together into one piece, and then to part or divide the ſame another way; namely, that the line of Diviſion ſhall goe from the point Q with a Right line in ſuch manner, that he which did owe the part M N O P ſhall have his part laid towards O, and the other his part towards P, and ſo divided that each may have as much of the whole as his part before was, that is, that the ſide towards O be equall to the Figure M N O P, and the part towards P ſhall be equall to the Figure P O Q R S.

To doe the ſame,

Let there be drawne from the point O a line that ſhall cut the point P (as before the ſame Land was divided) then draw the lines P N, P Q, and draw from O a line parallel to P Q cutting P N in the point H, then draw a line thorow that H point from Q to K in the ſide N M, then change or reduce the Figure M K H P into a Triangle, whoſe Baſe is K Q (or the ſide K Q, which you pleaſe to call it) and Q K M, which Triangle is K L Q. I ſay then that the line Q L is the line of Diviſion which ſhall divide the whole Figure in ſuch ſort that the part L N O Q is equall to M N O P and M N Q R S equall to P O Q R S, which was required.

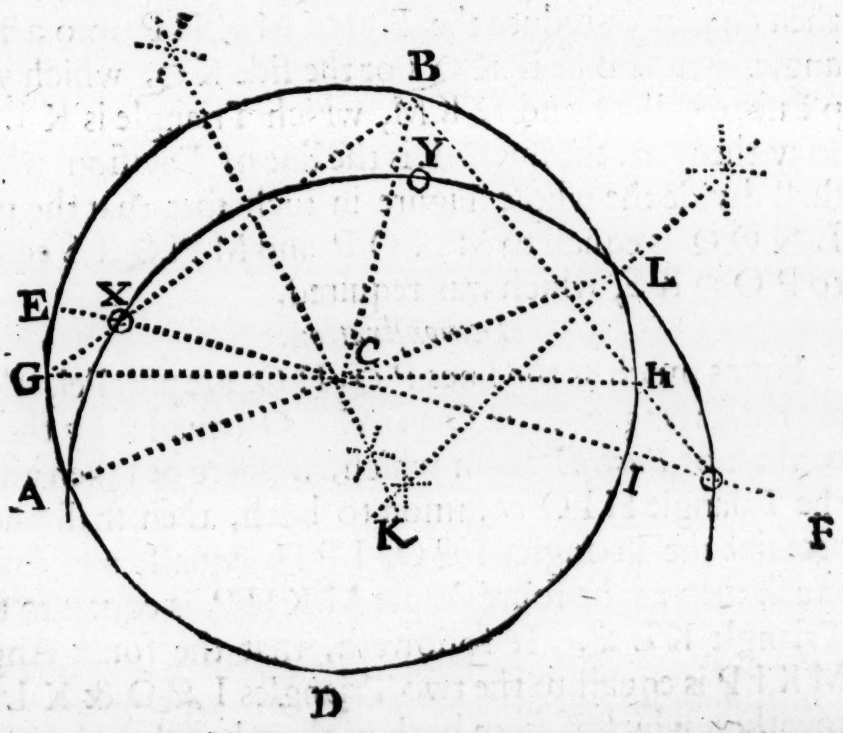
Demonſtration.

For as much as the lines P Q, H O, are parallel, then are the Triangles H Q O and H P O equall (by the 37 of the 1 of *Euclid*) from which, if there be taken away the Triangle H I O common to both, then ſhall there remaine the Triangles I Q O, I P H, equall, and for as much then as the foure Angle M K H P is equall to the Triangle K L Q. It followeth, that the foure Angle M K I P is equall to the two Triangles I Q O & K L Q together, which if from both of them be taken the com-

men piece KLCI. It appeareth then, that the four Angle MLGP is equall to the Triangle CQO, and the one being taken for the other, it is then evident that the part LNOQ is equall to the part MNOP, and that MLQRS is equall to the part POQRS. And if it were required that the line of Diviſion ſhould come out of any other Angle or point than from Q toward R, the ſame is here above ſufficiently ſhewed how the ſame may be done by paralel Lines either this way or that way.

Probleme XVII.

A Circle being given, and within the ſame Circle two points be taken, by adventure, by the ſame points given, to draw an Arch of a Circle, which ſhall divide the given Circles circumferences into two equall parts.



Conſtruction.

Conſtruction.

LEt the Circle given be A,D,H,B, the centre of the Circle is C, and the points taken by adventure within the Circle, as the points X and Y. It is required to draw an Arch of a Circle, which ſhall paſſe by the points X & Y, and cutting the ſame, alſo ſhall divide the ſaid Circles circumference in two equall parts.

To doe the ſame,

Draw a Right line from the circumference at E through the point X, and alſo through the centre of the Circle C to the circumference at I, and prolong the ſame right out infinite, then upon this Diameter E I from the centre C raiſe a Line perpendicular to the circumference in the point B, as the Right line C B, then from the point B and through the given point X to the circumference in G draw the Right line B X G, and then from that point in G and through the centre C draw the diameter G, C, H, cutting the circumference in the point H, then from the point B to the point of cutting in H draw the Line B H, and prolong the ſame till it ſhall interſect with the Line E I in the point F. And note that the point of cutting for your third point (then by the 25 of the 3 of *Euclid*) by thoſe two points given, and the third point found, as X, Y, and F, find the centre, which will fall in the point K, from which point, as the centre, draw the Arch A, X, Y, F. I ſay then, that this Arch ſhall divide the circumference of the given Circle, into two parts equall, in the points of interſection, A L, which was required to be done.

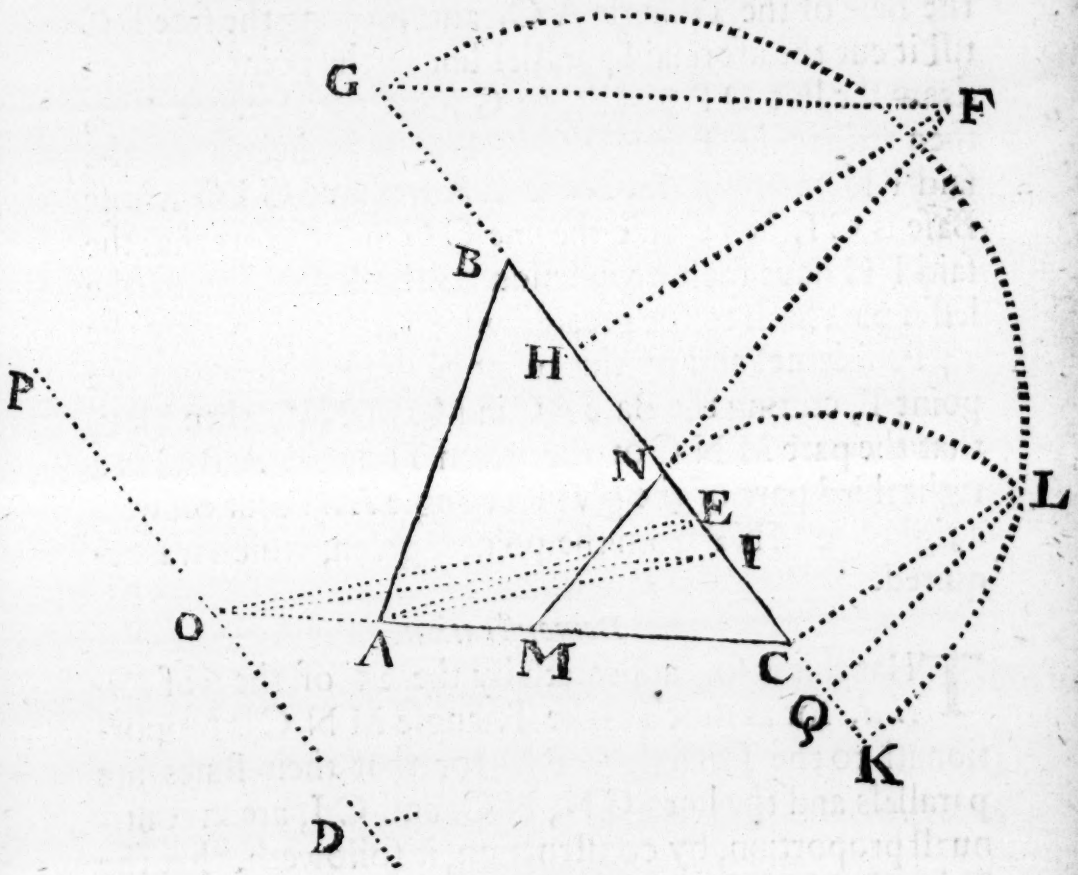
Demonſtration.

Demonſtration.

The Arch of the Circle (which divideth the circumference) A, X, Y, L, is drawne thorow the two given points X and Y, doth cut the circumference of the given Circle in the points A and L, and from the ſaid points A, L, is drawne the Right line A L, it is evident that if from the one point of cutting off the circumference of a Circle to the other be drawne a Right line, if the ſame line paſſe through the centre, then doth the ſame line divide the ſame Circle into two equall parts (by the 17 definition of the 1 book of *Euclid*) the Right line A L is drawne from the point A to the point L, and paſſeth through the centre C, it is therefore evident, that the ſame Right line A L doth divide the Circle into two equall parts, wherefore the Arch line A, X, Y, L, paſſing through the ſaid points at A and L, doth alſo divide the circumferences of the given Circle into two equall parts, which was to be demonſtrated.

Probleme XVIII.

A Triangle being given, and without the ſame Triangle a point being taken, how, with a Right line from the ſame point, to cut off from the Triangle any part required.



Conſtruction.

LEt the Triangle given be ABC, and let the point taken without the ſame Triangle be F. It is required, with a Right line, from the point F, to cut off from the Triangle A B C, a right third part of the whole Triangle.

To doe the ſame, divide the ſide of the Triangle (next to the point given, as in this, the ſide B C) into three equall parts, whereof one third part is C E, and from E draw an occult line E A, then is the Triangle C E A the Right third part of the whole Triangle ABC, then from the point F let fall a line perpendicular, as H F falling upon the ſide B C; then draw from F a line parallel to the Baſe of the Triangle A C, and prolong the ſide B C till it cut the aforeſaid parallel line in the point G, then draw the line D P parallel to G K of the diſtance F H, then reduce the Triangle C E A into the height of the ſaid F H, and ſo in ſtead of C E A we have C I O whoſe Baſe is C I, then divide the line C G in ſuch ſort that the ſaid F H have ſuch proportion to the leſſer part as the leſſer part hath to the greater, which let be C N (by the 14 Probleme) and by this point N draw a line from the point F, cutting the Baſe A C in M as F N M, then I ſay, that the part M N C of the given Triangle A B C is a right third part of the given Triangle ABC, cut off with a right line M N F from the point F given, which was required.

Demonſtration.

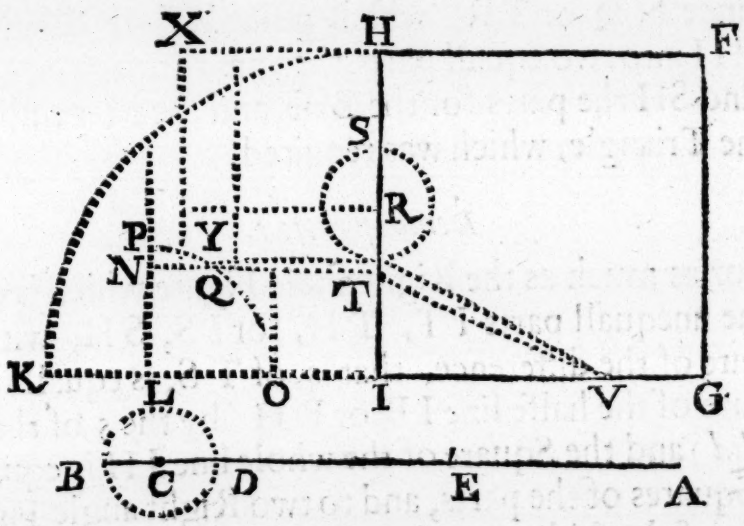
THat this is ſo, appeareth by the 25 of the 6 of *Euclid*, for as much as the Triangle M N C is proportionall to the Triangle G F N for that their Baſes are parallels and the lines G N, N C, and C I, are in continuall proportion, by conſtruction, it followeth, that the Triangle N M C which is made of the meane proportion, is equall to the Triangle O I C by reaſon of the two Triangles F G N and O C I, are of equall height, but the Triangle O C I is equall to the Triangle A E C, it followeth that M N C is alſo equall to A E C by the firſt common ſentence, and as A E C is one third part of

ABC

A B C, it followeth, that M N C is alſo one third part of the ſame A B C which was required to be done.

Probleme XIX.

To divide a given Right line into three parts, in ſuch ſort, that the ſame may forme a Right angled Triangle, or of the ſame parts may be made a Triangle, in which may be inſcribed a Circle, whoſe Diameter ſhall be alſo a line given.



Conſtruction.

LEt the given Right line be the line A B, and from the ſame line let there be cut off the Diameter of the Circle C, and the reſt of the line let be divided into two equall parts in E, as A E, and E D, and let one of thoſe parts be the Hypothenuſa of the Triangle, and then is the reſt of the line as E B the length of the Baſe and Perpendicular together; and now to know each of them apart, of this ſaid reſt E B, make thereof a ſquare, as F G I H, then with the diſtance I H from the

point I make the Arch H K and produce G I to K, and from I, upon the line I K, ſet the diſtance A E, comes to L, and from the point L raiſe a perpendicular L M, and divide the ſame LM into two equal parts in the point N, vide alſo I L into two equall parts in O, then from the point L, as a Centre, and with the diſtance O L, draw an Arch of a Circle, as P O, then from the point N draw the little meane proportionall N Q, which diſtance N Q ſet upon the line H I, from T to R, and from this Centre R, draw a Circle T S, whoſe halfe Diameter let be the diſtance N Q or T R, which point R doth divide the line H I into two equall parts, then are the unequall parts I S and SH the parts for the Baſe, and the Perpendicular of the Triangle, which was required.

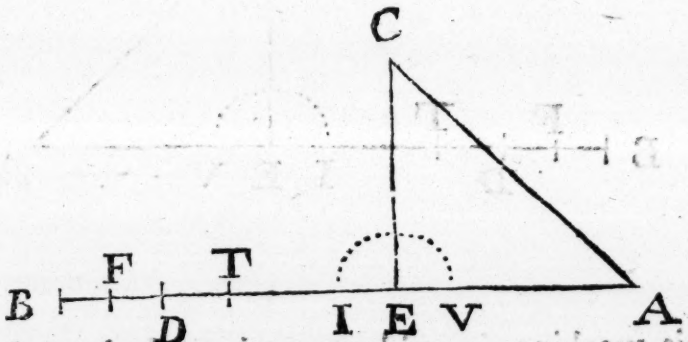
Demonſtration.

For as much as the Right angle Figure which is made of the unequall parts I T, T H, or I S, S H, with the Square of the difference, that is, of T S, is equall to the Square of the halfe line I R or R H (by the 5 of the 2 of *Euclid*) and the Square of the whole line I H is equall to the Squares of the parts, and to two Right angle Figures made of the ſaid parts (by the 4 of the 2 booke aforeſaid:) If therefore from the ſquare of H I be taken the ſquare of V T equall to the ſquares of the parts T I, I V, (by the 47 of the 1 booke of *Euclid*) there ſhall reſt the two Right angle Figures of the two ſaid parts: and if from the ſquare of the halfe line I H, namely the ſquare of H R, we take away the halfe of this reſt, it is evident that the reſidue is the ſquare of the difference T R, (by that which hath been ſaid before) by this it is alſo manifeſt that the ſquare of this Reſt being ſubſtracted from a fourth part of the ſquares of the parts, reſt the ſquare

of

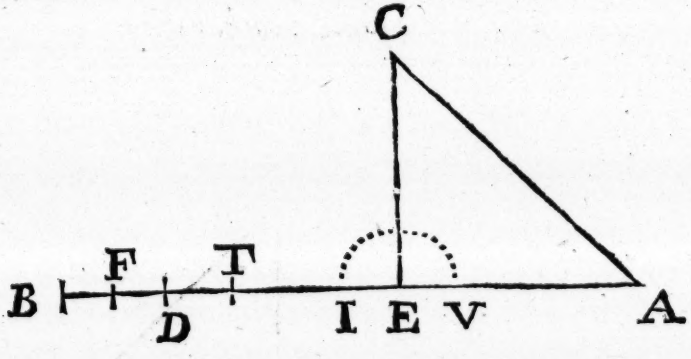
of the halfe difference; for, by the 9 of the 2 of *Euclid*, the ſquares of the unequall parts, as I S, S H, is double to the ſquare of the half line R H, and of the half of the difference R S, ſo that it inſueth from hence, as before, that the double of the ſquare of V T, which the ſquare of the parts exceedeth the quadruple of the ſquare of H R, foure times the ſquare of the halfe difference, or exceedeth the ſquare of H I, the quadruple of the ſquare of the difference, if therefore the ſquare of the halfe of L M be ſubſtracted from the ſquare of the halfe of V T, there ſhall the ſquare reſt of the ſection of the difference, which was required to be done.

To doe the ſame another way.



LEt the line given be A B, from which let there be cut off the part for the Diameter of the Circle as before was done, as B D, then the remainer of the line, as D A, let be divided into two equall parts in the point E, from which point E raiſe a perpendicular, as E C, equall to E A, and draw the line A C, which is equall to the part

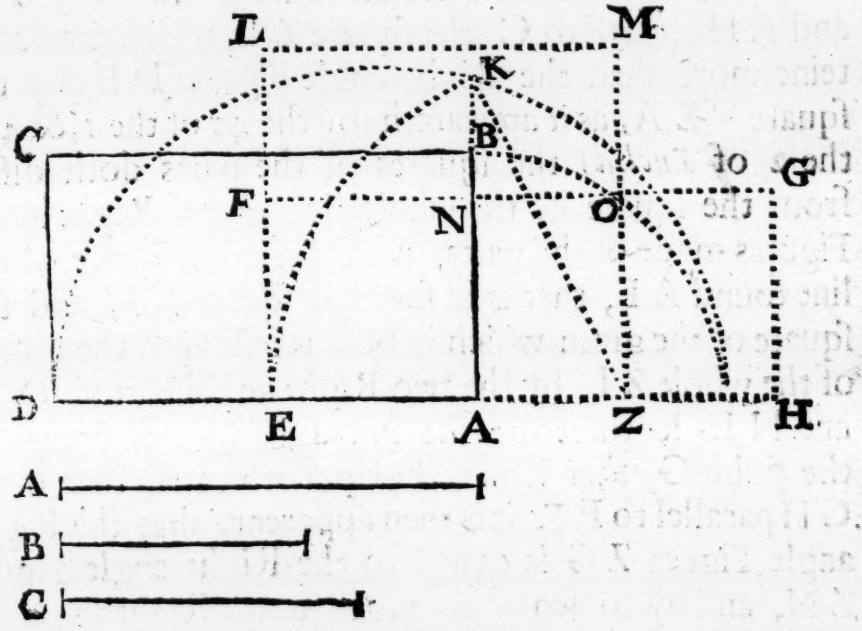
B E, which is alſo the length of both the ſides of this Triangle together, divide now this part B E into two equall parts in the point T, then is B T or T E the right halfe of both the ſides together; and now to finde the right part for each ſide, divide the diameter, cut off, as B D, into two equall parts in the point F (for the centre of the circle) and take the half of this diameter, as F B, or F D, and adde the ſame to T E, comes T V, the greater ſegment or ſide of the Triangle, or ſubſtract the ſame F D from T E (being equall to E I) reſt T I the leſſer ſide, which was required to be done:



This working is evident, and needeth no Demonſtration.

Probleme XX.

Three lines being given, to adde to one of them, a fourth line, in such sort, that the right Angle Figure made of the whole line, and the line added, as one line, and the line added shall be equall to the Right angled Figure, made of the two other lines.



Construction.

THe three lines given are A, B, C, to this line C it is required to adde such a line, as that the right angle Figure made of the same, and the line added as one line, and of the added line, be equall to the right angle Figure, made of the lines A and B.

To this end let there be made of the said lines A and B, the right angled Figure A B C D, the side then of a square, which is equall to the same, is A K (by the 23 Probleme)

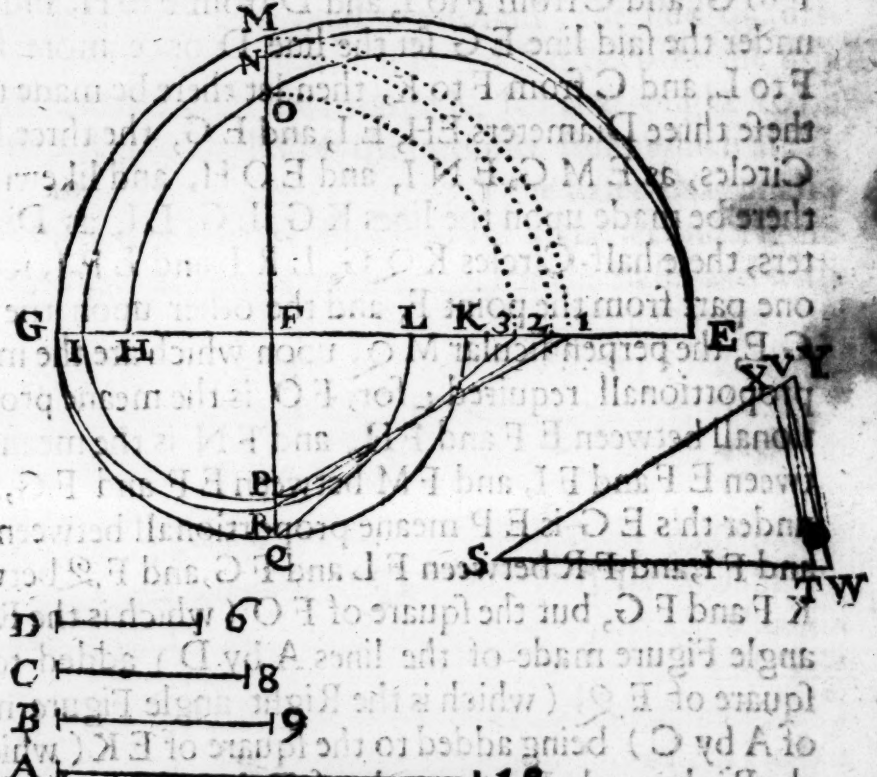
Probleme) and to the line A D is added the line C (by the 2 of the 1 of *Enclid*) which let be A H, divide the ſame A H into two equall parts, in the point Z, and with the diſtance Z K, having drawne the line Z K, make the Arch of a Circle K E, from the centre Z, then I ſay, that the line E A is the fourth line required.

Demonſtration.

Let there be made of the half of the given Lines A D, and A H (equall to C) the ſquare Z L, which ſhall conteine more than the Right angle Figure D B, by the ſquare of Z A, as it appeareth (by the 47 of the 1, & 4 of the 2 of *Euclid*) the ſquares of the parts doth differ from the ſquare of the whole line by two Right angle Figures made of the parts, wherefore, the ſquare of the line found A E, that is to ſay, the ſquare A F, and the ſquare of the given, which is N M, is leſſe than the ſquare of the whole Z L, by the two Right angle Figures O A, and N L; let therefore the line F N be prolonged to the point G, that it may be equall to A H, then draw G H parallel to E F, it is then apparent, that the Right angle Figure Z G is equall to the Right angle Figure Z N, and by conſequence equall to the Right angle Figure N L, by the firſt common ſentence, it followeth that the Right angle Figures G E and A C are equall, for as much as the ſquare of Z E doth exceed the Right angle Figure A C by the ſquare M N, and the ſame ſquare doth exceed the Right angle Figure H F, alſo the aforeſaid ſquare M N, which was required to be demonſtrated.

Problem XXI.

Of foure unequall Right lines to ferme a Quadrangle or Quadrilaterall, about which, a Circle may be circumscribed.



Construction.

LEt the foure lines be A, B, C, D, of which, it is required, to make a four-sided Figure, about which, a Circle may be circumscribed.

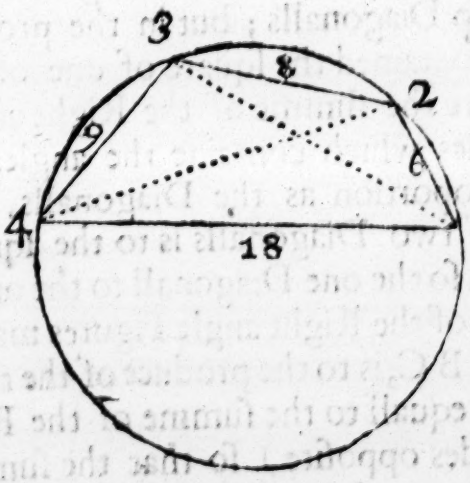
To doe this,

Seek first the meane proportionall Lines, as between A

and D, and between A and C, and between A and B, which meanes are F Q, F N, and F M, and alſo the meanes between B D and C D, which let be F P, F R, and F Q, which is done in this manner:

Let there be drawne a Line infinite, as G F prolonged forth, and upon the ſame let be ſet the line A, from E to F, and from the point F ſet the lines B, C, D, as B from F to G, and C from F to I, and D from F to H, and then under the ſaid line E G ſet the line D once more from F to L, and C from F to K, then let there be made upon theſe three Diameters E H, E I, and E G, the three halfe Circles, as E M G, E N I, and E O H, and likewiſe let there be made upon the lines K G, L G, L I, as Diameters, theſe half-Circles K Q G, L R I, and L P I, raiſing one part from the point F, and the other upon the line G E, the perpendicular M Q, upon which are the means proportionall required; for, F O is the meane proportionall between E F and F H, and F N is the meane between E F and F I, and F M between E F and F G, and under this E G is E P meane proportionall between F L and F I; and F R between F L and F G, and F Q between K F and F G, but the ſquare of F O (which is the Right angle Figure made of the lines A by D) added to the ſquare of F Q, (which is the Right angle Figure made of A by C) being added to the ſquare of E K (which is the Right angle Figure made of D by B, we have the ſquare of the line 2 K, and laſtly, the ſquare of F M (which is the Right angle Figure made of A by B) added to the ſquare of R F (which is the Right angle Figure made of C, multiplied by D) comes the ſquare of the line P I; therefore now to know the two Diagonals, let there be made the Angle W S X, and upon the line S W let there be firſt ſet the line 2 3, and alſo the line

R 2 from S to W, and upon the line S X ſet the line P 1 as from S to V, and drawing from the point W a Right line to V, and from the point T a Right line parallel to to the ſame as T Y, then is S Y the leſſer Diagonall; but if from the point T be drawne a Right line to V, and from W a Right line parallel to the ſame, as W X, then is S X the greateſt Diagonall; and now to deſcribe (by the help of theſe Diagonalls ſo found) the ſaid quadrilaterall, then marke of which lines I S is become the ſide of the ſquare P, ſet that above X, for that ought to be the two Right angles of the oppoſite ſides which are A by B, and C by D, and then is S Y the leſſer Diagonall; Firſt make the line 1,4, equall to A (in the ſecond Figure following) and upon the line 1,4, for the Baſe, and with



the diſtance S V or the greateſt Diagonall for the one, and the line D for the other ſubtending ſide make the Triangle 1,2,4, then upon the aforeſaid Baſe 1, 4, with the diſtance S Y, or the leſſer Diagonall 1,3 for the one, and the line B for the other ſubtending ſides make the Triangle 1,3,4, then remaineth the ſide 2, 3, for the line

C, which is common to both the aforeſaid Triangles in part, and may have either of the Diagonalls to his Baſe, and the ſubtending ſides may be two given lines, as 4 2, the Baſe and the Lines B and C the ſubtending ſides or the Diagonall 1 3 to the Baſe, and the lines C and D to the ſubtending ſides, and ſo we have the quadrilater prepared for the circumſcribing about the ſame, (which is done, by the 5 of the 9 booke of *Euclid.*)

Demonſtration.

The Demonſtration hereof is evident (by the 15 Theoreme of our ſecond part enſuing) as alſo by the 9 Chap. of the 1 booke of the *Almageſte* of *Ptolome*, that is, that the ſumme of the Right angle figure made of the ſides of the oppoſite angles, is equall to the Right angle Figures made of the two Diagonalls; but in the product of the Diagonalls is contained the ſquare of one of them, ſee there, wherefore the ſumme of the Right angle Figure made of the ſides which containe the angles oppoſite, are in ſuch proportion as the Diagonalls, that as the product of the two Diagonalls is to the ſquare of the one whole, even ſo the one Diagonall to the other, therefore the ſumme of the Right angle Figures made of C D, D A, and A B, B C, is to the product of the two Diagonalls (which is equall to the ſumme of the Right angle Figure of the ſides oppoſite) ſo that the ſumme of the Right angles Figures A B, A D, B C, C D, to the ſquare of the greateſt Diagonall A C, wherefore the worke is good: And becauſe the working ſhould be the better underſtood, we have here added numbers to the ſame, as followeth:

18	8	18	6	18	8
6	9	8	9	9	6
108	72	144	54	162	48

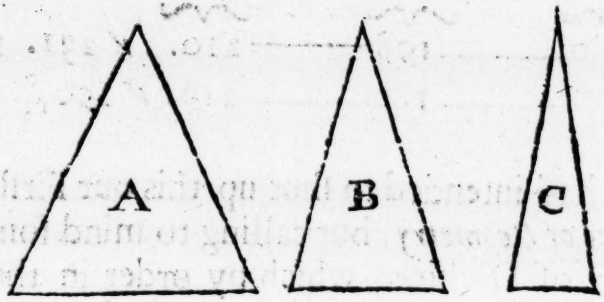
180——198——210. $\sqrt{231}$. 1. Diag.

198——180——210. $\sqrt{190\frac{10}{11}}$. 1. Diag.

Here we had intended to shut up this our First part of the *Practice of Geometry*, but calling to mind some other Problemes of Division, which by order in the worke should have had their place about the 37 Probleme, But rather than to deprive the young Practiser of the use of such pleasant variety, we thought good to set them downe here than wholy to omit them: These are in number foure.

Probleme XXII.

To divide a Figure given, according to the proportion of two or more Figures given, after what manner ſoever.

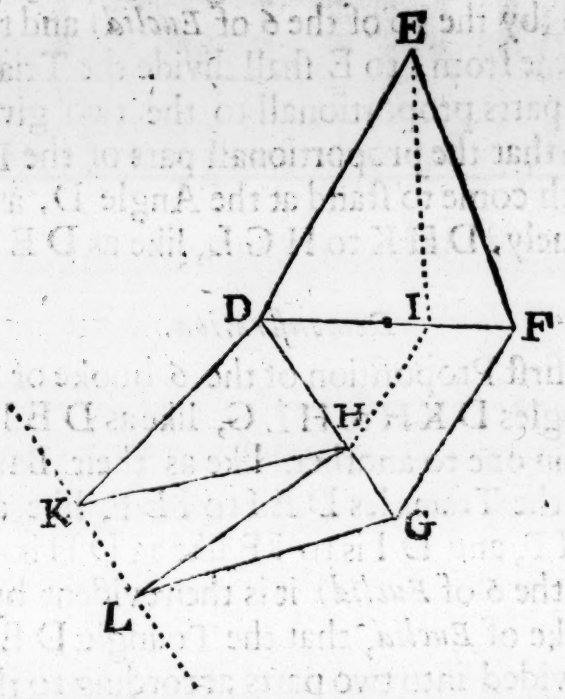


Conſtruction.

LEt the Figures given be of any forme whatſoever, and being reduced into Triangles of one height, (by the 44 Propoſition of the 1 booke of *Euclid*) which let be theſe three Triangles, A, B, C, then are thoſe Figures in proportion the one to the other, as their Baſes, according to which proportion the Baſes or other ſides which here before are divided in equall parts ſhall be divided (by the 10 of the *6* of *Euclid*) and then forth, doing in all things as before in Diviſion is ſhewed. Of which working we ſhall uſe but one onely example, to ſhew the practice of proportionall Diviſion: Thus, Let there be three Right line Figures given, which being reduced into Triangles of equall height, as here above; it is now required to divide the Figure quoted A into two parts, ſo that theſe parts ſhall have proportion one to another, like as the Figures quoted B and C, hath the one to the other.

To

To doe the ſame,



Set downe the Triangle which ſhall be divided, before quoted A, and in this is the Triangle D E F, then from which Angle you would have the proportion of the Figure B have this place cut off from the ſaid Triangle (as in this example it is required from A) draw a Right line, making an Angle with the Baſe D E, either a ſharp or blunt Angle, at your pleaſure, as in this the line G D, and upon this line D G ſet the Baſe of the Triangle of the ſecond Figure, quoted B, which Baſe comes to H, and at the point H joyne the other Baſe of the leſſer Figure, quoted C in the ſame Right line, which comes to G, then is the whole line D G equall to the two Baſes of the Figures quoted B and C, this done, draw a Right line from G to F, and another parallel to the ſame from the point H cutting the Baſe D F in the point I, then is the

the Baſe D F divided according to the proportion of the line D G (by the 10 of the 6 of *Euclid*) and then a Right line drawne from I to E ſhall divide the Triangle D E F into two parts proportionall to the two given Figures, & ſo alſo that the proportionall part of the Figure quoted B doth come to ſtand at the Angle D, as was required; namely, D H K to H G L, like as D E I to D E F.

Demonſtration.

By the firſt Propoſition of the 6 booke of *Euclid*, are the Triangles D K H to H L G, like as D E I to I E F in proportion one to another, like as their Baſes D H to H G and the Triangles D E I to I E F, like as the Baſes D I is to I F, and D I is to I F, like as D H is to H G (by the 10 of the 6 of *Euclid*) it is then evident by the 11 of the 5 booke of *Euclid*, that the Triangle D E F is by the line I E divided into two parts according to the proportion of the Triangles D K H to the Triangle H L G, and conſequently according to the proportion of the given Right line Figures.

The End of the I Part.

A HUNDRED GEOMETRICAL QVESTIONS,

WITH

Their Solutions and Demonſtrations:
Moſt of them being reſolved both Arithmetically and Geometrically, by Lines and Numbers, &c.

By Captain THOMAS RUDD,
Chiefe Engineer to His late Majeſty.

LONDON,
Printed by ROBERT LEYBOURN,
for ROBERT BOYDELL, at the Bulwark neere Tower-Hill, *MDCL.*

TO THE READER.

FRIENDLY READER:

Ere is preſented unto thee a hundred Geometricall Queſtions, *and alſo Inſtructions how to anſwer and give a right ſolution to every of them, ſome by Lines, and ſome by Numbers,*

(yet all without Algebra:) *very neceſſary for all thoſe who will have to doe with* Geometry, *In regard, that herein is handled and propounded the chiefeſt and principalleſt parts of Menſurations belonging to the ſame Art; How to perform all manner of Meaſurings very artificially and ſpeedily; and how to finde out Lines which be hidden or ſecret in the ſame: To the great eaſe and pleaſure of the Practiſer, and alſo very delightfull to all men.*

A Hundred GEOMETRICAL QVESTIONS.

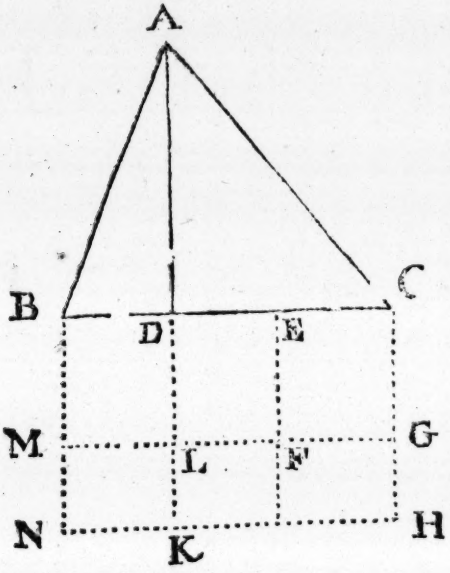
QUESTION 1.

There is a Triangle, as A B C, *whereof the ſide* A B *is* 13, *and* A C *is* 15, *and* B C *is* 14.

The Queſtion is; *How long is the Perpendicular Line* A D?

TO know this, *Note*, That if the Square of A B be ſubſtracted from the Square of A C, there ſhall then even as much remain, as if the Square of B D were taken from the Square of D C. (For as much as the Squares of A D and D B together, are equall to the Square of A B: and alſo the Squares of A D and D C are together equall to

the Square of A C, *by the 47 of the 1 of Euclid.* Now the Square of A B is 169, and the Square of A C is 225: take therefore that 169, from 225, and there ſhall reſt 56 for the Gnomon D E F G H K D, which Gnomon is equall to G M N H, by reaſon that D E F L is

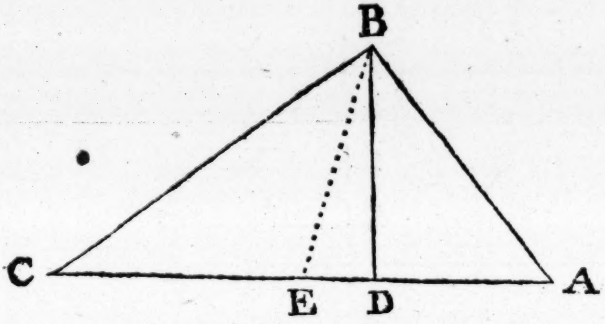


equall to L K N M. Then is M G N H alſo 56: and ſeeing the length N H or B C is known to be 14, therefore divide the ſame 56 by 14, the Quotient is 4 for N M equall to D E; ſubſtract D E 4, from B C 14; reſts 10 for B D and E C together; and ſeeing that B D and E C are equall, the halfe of 10 is 5 for B D, *and by the aforeſaid 47 of the 1 of Euclid,* is A D known, by ſubſtracting the Square of B D 25 from the Square of A B 169, reſts for the Square of A B 144, whoſe Square Root is 12 for A D, which was required.

QUEST. 2.

In this Triangle A B C, *the angle* B *is a right angle, whoſe perpendicular* D B *is unknown, but the parts of the baſe where the ſame ſhall fall is known, for* A D *is* 18, *and* D C *is* 32.

The Queſtion is; *How much is the perpendicular line* B D?



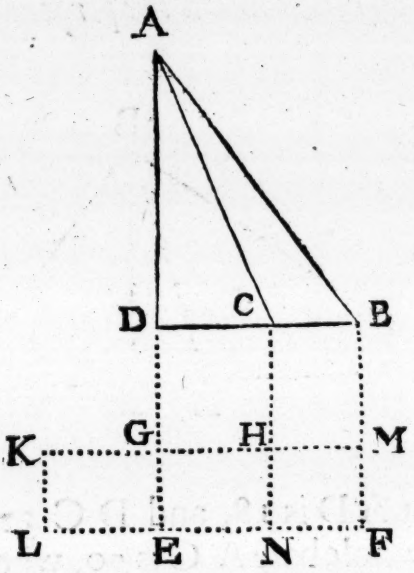
SEeing that A D is 18, and D C 32, it followeth that the whole baſe A C is 50, whoſe halfe is 25, for A E or E C (or for E B, becauſe the angle B is a right angle) therefore, take A D 18, from C D 32, reſts 14, whoſe halfe is 7 (or take 18 from 25, the halfe of 50) reſts 7 for the difference D E. And ſeeing now that B E can be known, for that A E, E C, and E B are equall (by reaſon that B is a right angle) that is, each of them 25, therefore *by the 47 of the 1 of Euclid*, can B D be alſo known, by ſubſtracting the ſquare of E D 49, from the ſquare of B E 625, reſts the ſquare of B D 576, whoſe ſquare Root is 24 for B D, the perpendicular line required.

QUEST.

Quest. 3.

There is a Triangle A B C, *whose three sides are known,* A B 15, A C 13, *and* B C 4.

The Question is; *How much is the perpendicular line* A D, *which falleth without the Triangle in the point* D?



THe square of A D and the square of D B together, are equall to the square of A B, *by the 47 of the 1:* Also the squares of A D and of D C are together equall to the square of A C. *Now,* If we substract the square of A C from the square of A B, there shall remain as much as if we substract the square of D C from the square of D B.

Now, The square of A B is 225, and the square of A C is 169, which taken from 225 rests 56, and so much is the square of A B greater then the square of

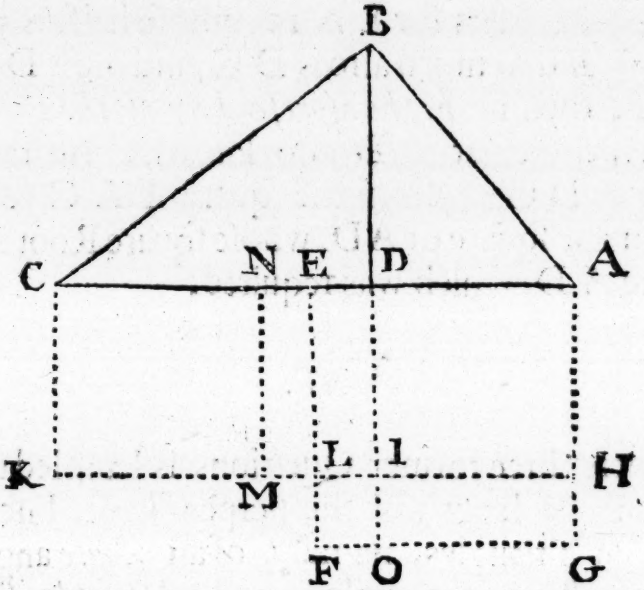
of A C, or the ſquare of B D, more then the ſquare of C D, and this 56 is equall to the Gnomon C B F E G H C, and alſo equall to the Parallelogram K M F L: For as much as C M is equall to L G, therefore is K M F L alſo 56, and C B being known to be equall to K L, and is 4, therefore divide 56 by 4, the Quotient is 14 for F L or K M, from which ſubſtract F N 4, equall to C B or K L, reſts for L N 10, whoſe halfe is 5 for E L or E N, and ſo much alſo is D C, and now D C being known, then is *(by the aforeſaid 47 of the 1 of Euclid)* the perpendicular A D alſo known, by taking the ſquare of D C 25, from the ſquare of A C 169, reſts 144 for the ſquare of A D, whoſe ſquare Root is 12, for the line A D, which was required.

Note.

IN theſe three former Queſtions is handled the finding of the length of the perpendicular line in three kindes of Triangles, namely, of an *Acute* angled, an *Orthogonall* or right angled, and of an *Obtuſe* angled Triangle, their three ſides being known; we have found their perpendiculars by Lines, we ſhall likewiſe proceed in finding the ſegments or parts of the Baſe cut off by the ſaid perpendicular line falling from the upper angle, by lines alſo, and then ſhew how to finde the ſame by Numbers in all theſe kindes.

QUEST. 4.

There is a Triangle A B C, *rightangled in* B, *whereof the Baſe* A C *is* 50, *and the perpendicular* B D *is* 24. The Queſtion is; *How much is the parts of the Baſe* A D *and* D C *each apart?*



THe ſquare of B D is as much as if we multiply A D by D C, and therefore is a right-angle Figure made of A D and A C equall to the ſquare of A D, and more 576, take the halfe of A C comes A E 25, whoſe ſquare is 625 for A E F G, and from hence take D I K C, that is 576, there doth reſt the ſquare I L F O, that is 49; aud becauſe H I O G is made equall to L E M N, therefore is the Gnomon A G O I L E A alſo equall to D I K C 576; if then the ſquare I L F O be 49, then is the line O F 7, and ſo

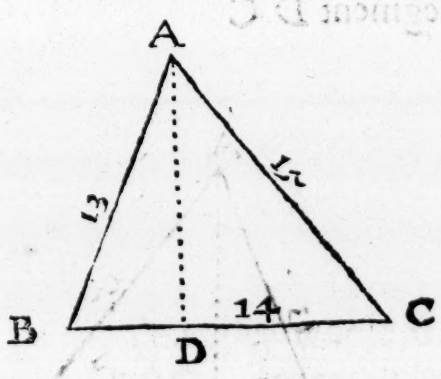
ſo much is alſo D E the difference, which being taken from A E 25, reſts 18 for A D, and added to C E 25, comes 32 for D C, the parts and ſegments of the Baſe, which was required.

How by Numbers to work theſe former Queſtions: A generall Rule.

ADde the ſquare of the Baſe and the ſquare of the leſſer ſide, and from that ſumme ſubſtract the ſquare of the greater ſide, and the reſt divide into two equall parts, and one of theſe parts divide again by the baſe, the Quotient thereof gives the leſſer ſegment, which being ſubſtracted from the whole baſe, there ſhall remain the greater ſegment.

EXAMPLE.

By the three ſides of a Triangle, to finde the parts of the Baſe, and alſo the Perpendicular.



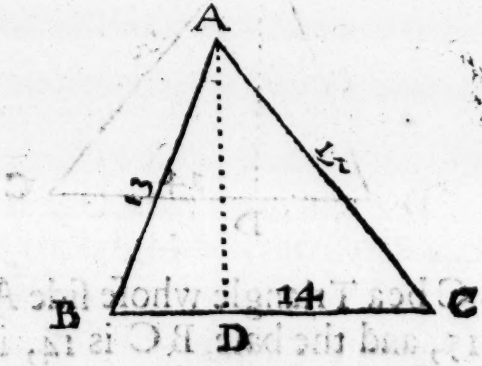
LEt A B C be a Triangle whoſe ſide A B is 13, and A C is 15, and the baſe B C is 14, in which Tri-

angle is drawn the perpendicular line A D, falling upon the Baſe B C within the Triangle, dividing the ſame Baſe into two parts: And it is required to finde how much each of theſe two parts of the Baſe C D and D B are, and alſo how much is the perpendicular line A D.

CONSTRUCTION.

The Order of the Numbers.

The ſquare of the whole baſe B C 14, is	196
The ſquare of the ſide A B 13, is	169
Which being added, comes	365
From which ſubſtract the ſquare of the ſide A C 225, reſts	140
Whoſe half is	70
Which divided by the baſe B C 14, comes for the leſſer ſegment B D	5
Which ſubſtracted from B C 14, reſts for the greater ſegment D C	9

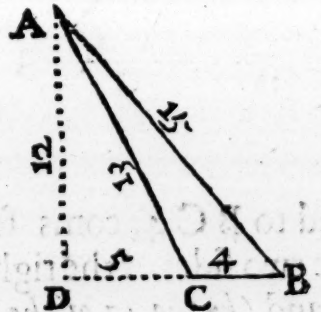


And

And now ſeeing that the Triangles are right angled, namely A D B, and alſo A D C, whereof two ſides of each are known, that is, the ſides A B 13, and B D 5, as alſo A C D, the ſide A C 15, and C D 9: then *(by the 47 of the 1 of Euclid)* is the third ſide or perpendicular alſo known; for the ſquare of A B is 169, from which take the ſquare of B D 25, reſts 144, whoſe ſquare Root is 12 for A D.

Or otherwiſe: The ſquare of A C is 225, from which ſubſtract the ſquare of C D 81, reſts 144 for A D whoſe ſquare Root is 12 for A D as before: and by this means are not only the parts or ſegments of the baſe, but alſo the perpendicular of the Triangle known, which was required.

In oblique angled Triangles, where the perpendicular falleth without the Triangle, found alſo by Numbers.

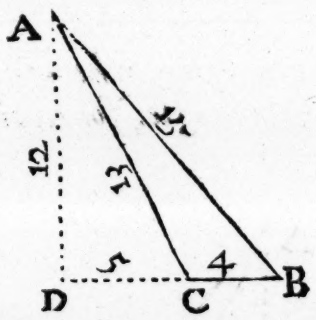


LEt A B C be the Triangle, whoſe ſide A B is 15, and A C is 13, and the perpendicular A D being drawn doth fall without the Triangle, upon the produced baſe B C in the point D, making of the baſe two

parts, D C and C B: It is by Numbers required to finde how much are those two parts of the base D C and C B each apart; and also the perpendicular of the Triangle A D.

Construction.

THe square of the base B C is 16, to which adde the square of the lesser side 13, as 169, comes 185, this 185 substracted from the square of the greater side, as from 225, rests 40, whose halfe is 20, which being divided by 4 the lesser side, comes 5, for the prolonged



base, which added to B C 4, coms for the whole base B D 9. Then are two sides of the right angled Triangle A D B known, and *(by the 47 of the 1 of Euclid)* is the third side known, which is 12 for the perpendicular A D, which was required.

Demon.

Demonſtration.

IN the firſt of theſe two Queſtions wrought by numbers, the ſquares of A B, and of B C together, are greater then the ſquare of A C, by two right angle figures comprehended under the Lines B C and B D; *(by the 13 of the 2 of Euclid,)* but thoſe two ſquares are greater then the ſquare of A C by 140, therefore the two rightangle Figures compriſed under C B and B D, are together 140, and by conſequence, one right angled figure of C B and B D containeth the halfe of 140, that is 70, but the one ſide of this right angled Figure is known, and makes 14, therefore divide 70 by 14, and the Quotient is 5 for the other ſide of this Figure B D, or for the leſſer part of the baſe B D, which taken from 14 the whole baſe, reſts 9 for the greateſt part or ſegment of the baſe.

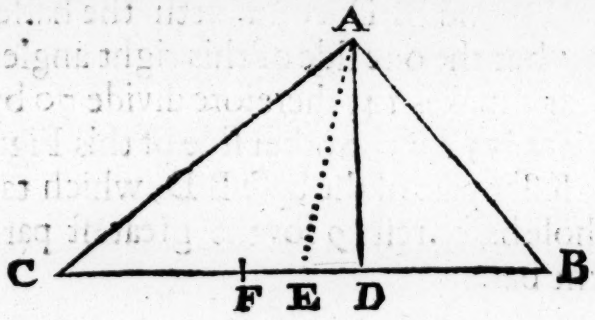
Thus have we ſhewed both by Lines and Numbers, How by the known ſides of any Triangle, to finde the unknown perpendicular falling within or without the ſame, and alſo the parts of the baſe cut off by the ſame perpendicular: The like may alſo be done by diverſe other Lines more, as in the following Queſtions will appeare.

QUEST.

Quest. 5.

There is a Triangle A B C, *right angled in* A, *whereof the two sides* A B *and* A C *are unknown, but the perpendicular* A D *is known, and is* 24, *also it is known that the one part of the base, as* D C, *is more then the other part* D B *by* 14.

The Question is; *How much are the three sides of the Triangle each apart?*



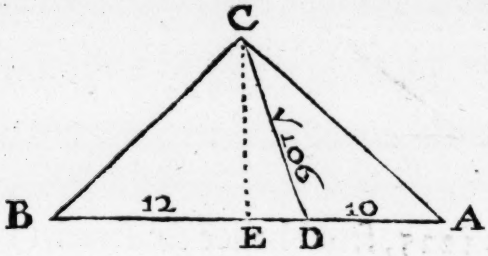
LEt F C be made equall to B D, then is D F 14, the halfe thereof is 7 for D E, and E is the midst between B and C, this square added to the square of A D, as 49 to 576, comes 625 for the square of A E, whose square Root is 25 for the line A E, and so much also is B E or E C. Then substract D E 7, from B E 25, rests 18 for D B, then is D C 32. And having the base and the perpendicular known, the other two sides are known *by the 47 of the 1 of Euclid*, and there shall come for A B 30, and for A C 40, which was required.

Quest.

QUEST. 6.

There is a Triangle A B C, *whereof* A C *and* C B *are equall, in this Triangle is a line drawn, as* C D, *long* √106, *ſo that* A D *is* 10, *and* D B 12.

The Queſtion is; *How much is the perpendicular line* C E?



ADde 10 to 12 comes 22, for the whole line A B, whoſe halfe is 11, for A E or E B, from hence ſubſtract 10 for A D, reſts 1 for D E, whoſe ſquare is 1, that taken from the ſquare of C D, as 106, reſts yet 105 for the ſquare of E C, whoſe ſquare root is √105 for the line E C, and then are the ſides A C and B C known, *by the 47 of the 1 of Euclid.*

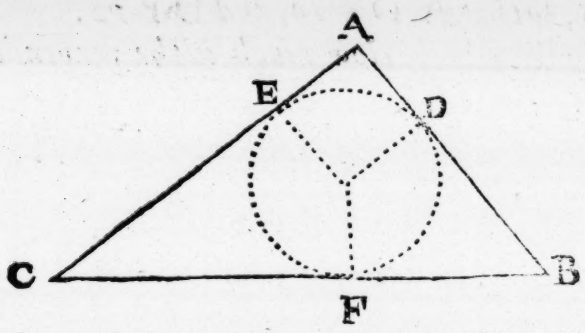
QUEST. 7.

There is a Triangle A B C, *right angled in* A, *the ſum of the 3 ſides together is* 120, *& the area or content is* 600.

The Queſtion is; *What the three ſides are each apart?*

DOuble the content comes 1200 which divide by 120 comes 10 for A D or A E, then is E A D 20, this ſubſtracted from 120, reſts 100 for B D, C E, the halfe is 50 for B C, and ſo much is B D

and C E together; this 50 taken from 120 reſts 70 for A B and E C together: and then to finde A B and A C apart, multiply the halfe of 70 (that is 35) in him-

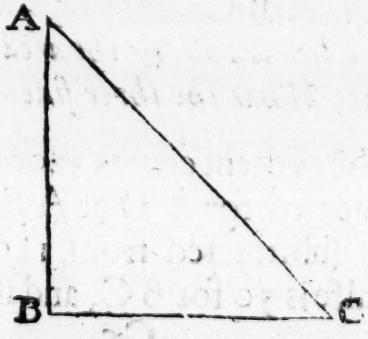


ſelfe, comes 1225, from hence ſubſtract 1200, reſts 25, whoſe ſquare Root is 5, which taken from 35 reſts 30 for A B, and added to 35 makes 40 for A C, and ſo are all the three ſides found, which was required.

Quest. 8.

In this Triangle A B C, B *is a right angle, of which it is known that the ſum of all the three ſides is* 4 + √ 8.

The Queſtion is; *If the ſides* A B, B C *be equall, how much is each ſide?*



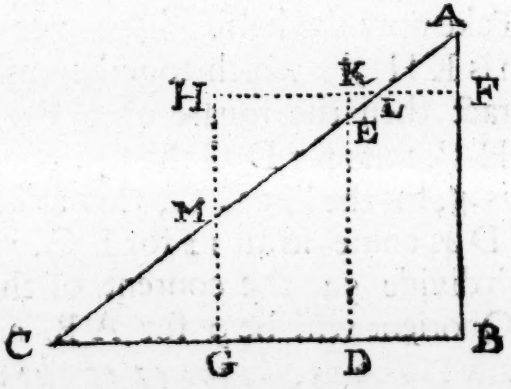
Sup-

SUppoſe that A B and B C be each 1, then is A C $\sqrt{2}$, and then the ſum of the three ſides ſhould be $2+\sqrt{2}$: then as $2+\sqrt{2}$ is to $4+\sqrt{8}$, ſo is 1 to the ſide A B or B C; ſay therefore, If $2+\sqrt{2}$ gives $4+\sqrt{8}$, what gives 1? and there ſhall come 2 for A B or B C, then is A C $\sqrt{8}$.

QUEST. 9.

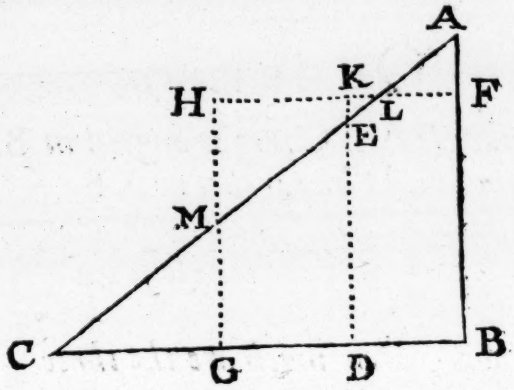
There is a Triangle A B C, *right angled in* B, *in which is drawn the line* D E *parallel to* A B, *ſo that* D B *is* 4, *and the content of the quadrangular figure* ABDE *is* 30, *and the reſt of the Triangle ſo cut off, as* E D C, *is* 24.

The Queſtion is; *How much are the three ſides of this Triangle?*



REduce the part A B D E into a Quadrangle, whereof the breadth is B D, the length is then B F or D K, and is $7\frac{1}{3}$, (which is known by dividing 30 by 4) and ſo is B F K D equall to A B D E, make alſo K D G H equall to E D C, being alſo 24, which is done by dividing 24 by $7\frac{1}{3}$, and there comes $3\frac{1}{3}$ for D G or K H, then is the Quadrangle B F H G

equall to the Triangle A B C and the Triangles A F L and G M C are both together equall to the Triangle L M H, and seeing those Triangles A F L, L H M, and M G C are all equiangled, therefore are they like pro-



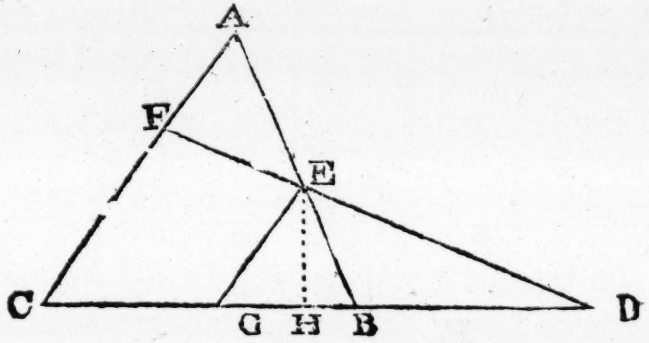
portioned. Take now the halfe of 4, comes 2 for L F or L K, then is K H $3\frac{3}{7}$, which together makes $\frac{1}{7}$ for L H. Subſtract then the ſquare of L F 2, from the ſquare of L H $\frac{3}{7}$, reſts for D G the ſquare $21\frac{4}{7}$, whoſe ſquare root is $4\frac{4}{7}$, for the line G C, this added to D G $3\frac{3}{7}$, and to B D 4, comes in all 12 for B C, whoſe halfe is 6, by which divide 54, the content of the Triangle A B C, the Quotient will be 9 for A B, and then is A C alſo known to be 15, *by the 47 of the 1 of Euclid*, which was required.

QUEST.

QUEST. 10.

There is a Triangle A B C, *whoſe ſides are* A B 13, B C 14, *and* A C 15, *whereof* B C *is extended to* D, *and ſo* B D *is* 10, *and from* D *is a line drawn through* E *to* F, *ſo that* B E *is* 5.

The Queſtion is; *how much* A F *and* F C *ſhall be each apart.*

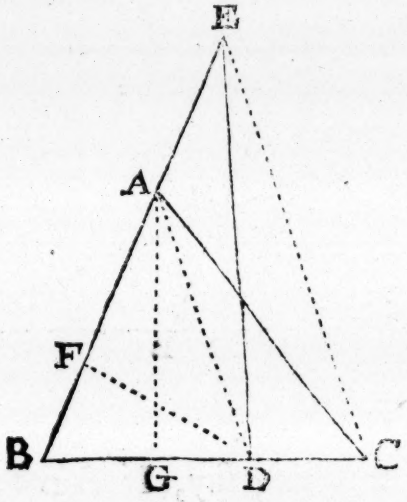


TO perform this, ſay, If A B 13, gives E B 5, what ſhall B C 14 give? and there ſhall come $5\frac{5}{13}$ for B G: then ſeeke E G, ſaying, If A B 13, give E B 5, what ſhall A C 15 give? and there comes $5\frac{10}{13}$ for E G: Then is the Triangle D E G of like proportion to D F C, therefore ſay again, If D G $15\frac{10}{13}$, gives E G $5\frac{10}{13}$, what ſhall D C 24 give? and there ſhall come 9 for F C. Then is A F 6, and E D and E F is alſo eaſily found,

QUEST. 11.

There is a Triangle A B C, *whose ſides are* A B 26, B C 30, *and* A C 28, *of which the line* A B *is prolonged to* E, *and then a line drawn from* E *to* D, *that is the line* E D, *ſo that the area or content of the Triangle* B E D *be equall to* A B C.

The Queſtion is; *If* B D *be* 18, *how much ſhall be* E B *and* E D *each apart.*



HEre muſt be underſtood, that theſe Triangles being equall, that is A B C to E B D, that the line A D is parallel to E C, and that the Triangles A E D and A C D are alſo equall, and ſtand between two parallel lines, and upon one baſe A D, wherefore thoſe Triangles A B D and E B C are of equall or like proportion, therefore ſay, If B D 18, giveth B A 26, what ſhall B C 30 give? and there coms $43\frac{1}{3}$, for B E: Then ſee how much is G B, and there ſhall come $13\frac{1}{3}$ for B G, then ſay, If A B 26, give B G

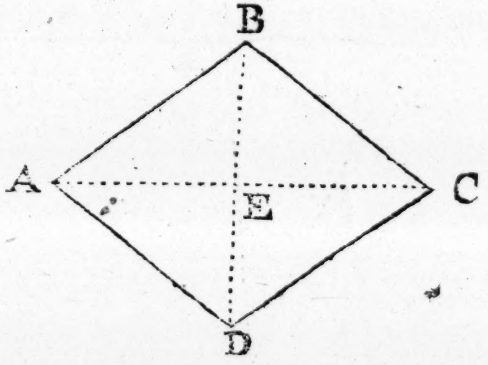
$13\frac{1}{3}$,

$13\frac{1}{17}$, what ſhall B D 18 give? and there ſhall come $9\frac{9}{17}$ for B F, this taken from B E $43\frac{1}{17}$, reſts for F E $34\frac{8}{17}$: then the ſquare of B F taken from the ſquare of B D, reſts for the ſquare of F D $240\frac{204}{289}$, to this add the ſquare of F E, comes for the ſquare of D E $1409\frac{1183}{1521}$, then is the line D E $\surd 1409\frac{1183}{1521}$, which was required.

QUEST. 12.

There is a Quadrangle whoſe four ſides are equall, as A B C D, *each ſide is* 10, *but the Diagonall lines are unequall, whereof* A C *is* 16, *for the longeſt.*

The Queſtion is; *How much is* B D *the ſhorter Diagonall?*



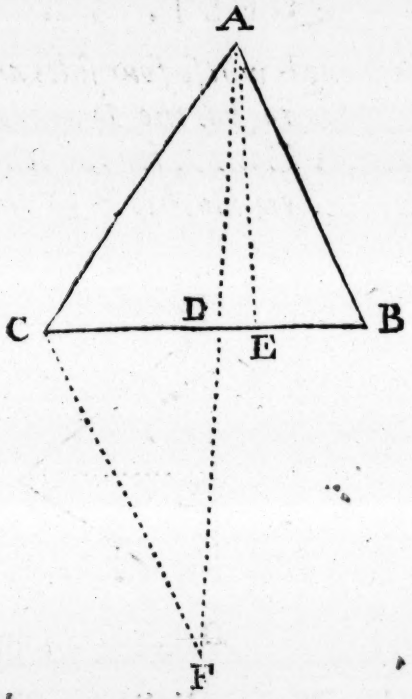
DIvide the Diagonall lines each in two equall parts, making right angles in the center, then becauſe A C the longeſt is 16, the halfe is 8, for A E or E C, then multiply 8 in it ſelfe comes 64, for the ſquare of A E, ſquare alſo one of the ſides 10, comes 100, ſubſtract 64 the ſquare of 8, from 100 the ſquare of 10, reſts 36, whoſe ſquare root is 6 for B E or E D, making for the whole Diagonall B D 12, which was required.

QUEST.

Quest. 13.

There is a Triangle A B C, *the sides whereof are* A B 13, B C 14, *and* A C 15: *In the same Triangle is a line drawn* A D, *which doth divide the angle at* A *into two equall parts.*

The Question is; *How much is that Line* A D?



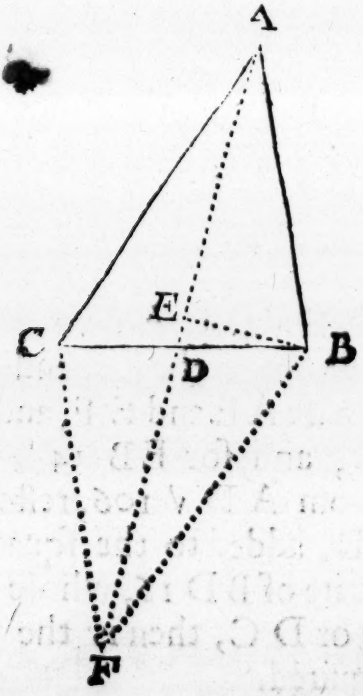
DRaw the line C F parallel to A B, and produce A D to F, then is C F equall to A C, because the Triangle B D A is of one proportion to the Triangle C D F, wherefore the angle D F C is equall to the angle D A C, and the line C F is equall to A C, and the line F C is to the line C D, as A B to B D: Then adde F C 15 to A B 13, comes 28, then say, If

28 gives the whole line B C 14, what gives 13? and there ſhall come $6\frac{1}{2}$ for B D; or ſay, what ſhall A C 15 give, by the firſt queſtion? and there comes $7\frac{1}{2}$ for D C, Then ſeeke B E and E A, and there comes for B E 5, and A E 12, ſubſtract now B E 5 from D B $6\frac{1}{2}$, reſts for E D $1\frac{1}{2}$, this ſquare added with the ſquare of AE, comes for the ſquare of A D $146\frac{1}{4}$, then is the line A D $\sqrt{146\frac{1}{4}}$, which was required.

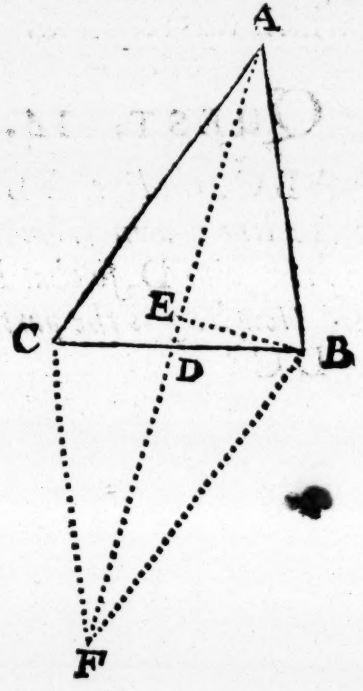
QUEST. 14.

In this Triangle A B C, *the line* A B *is* 10, *and* A C *is* 12, *and there is a line drawn from* A *to the midſt of the line* B C *in the point* D, *ſo that* A D *is* $\sqrt{106}$.

The Queſtion is; *How long is the parts of the line* B C, *that is,* B D *or* D C?



PRoduce the line A D as long again as it is, as A D F, then is A F √424, and draw B F and C F, then is B F 12, and C F, becauſe B D and D F are equall to D C and C A, having equall angles, then have we a Triangle as A F B, whoſe three ſides are



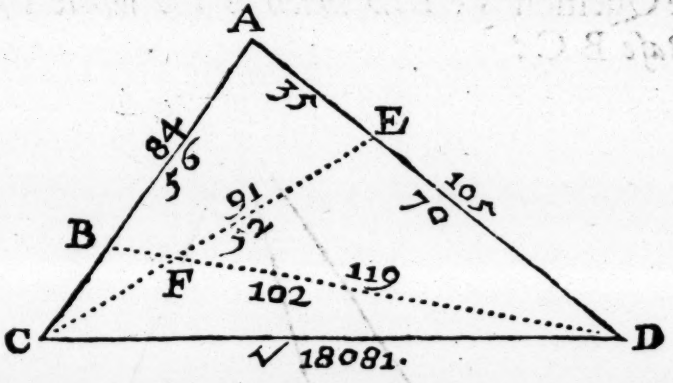
known, that is A F √424, A B 10, and B F 12; then (by the firſt Queſtion) get the perpendicular E B, as alſo the parts of the baſe A E and E F, and there ſhall come for A E √ $85\frac{15}{106}$, and for E B $14\frac{91}{106}$; ſubſtract then A E √ $85\frac{15}{106}$, from A D √ 106, reſts √ $1\frac{15}{106}$ for E D, this ſquare of E D, added to the ſquare of E B $14\frac{91}{106}$, comes for the ſquare of B D 16, whoſe ſquare root is 4. for the part B D or D C, then is the whole line B C 8, which was required.

QUEST

QUEST. 15.

There is a Triangle, as A C D, *whoſe ſides are known,* A C *84,* A D *105, and* C D *is* √ *18081: In which Triangle there are two lines drawn,* B D *being 119, and* E C *91, and theſe lines doe croſſe one another in the point* F, *and the ſaid lines doe cut the ſides of the Triangle in ſuch manner, that* A B *is 56, and* A E *is 35.*

The Queſtion is; *How much or how far the point* F *is diſtant from* B *or* E?



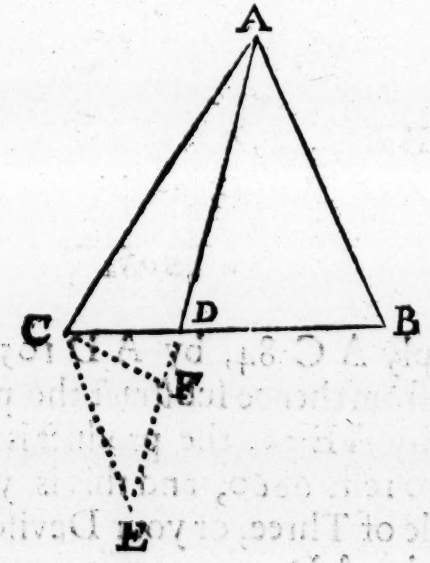
MUltiply A C 84, by A D 105, comes 8820, and from thence ſubſtract the product of B A 56, by A E 35, the product is 1960, that taken from 8820 reſts 6860, and this is your firſt number in the Rule of Three, or your Deviſor. Then multiply C B 28, by A D 105, comes 2940, this is your ſecond number in the Rule, and for the third number ſet the line C E 91, then multiply and divide, and there ſhall come in the Quotient 39 for the part C F, then is F E 52. And for the parts of B D take the former

Diviſor 6860 for your firſt number, then multiply D E 70, by A C 84, the product is 5880 for your ſecond number in the Rule, and for your third number ſet the line B D 119, then multiply and divide, and the Quotient is 102 for the part D F, which ſubſtracted from 119 the whole line, reſts 17 for B F, which was required.

QUEST. 16.

There is a Triangle as A B C, *whoſe two ſides are known,* A B 13, A C 15, *and from the angle* A *is a line drawn to the point* D, *upon the Baſe* B C, *which line is long* $\sqrt{162}$, *ſo that* C D *is* $\frac{1}{3}$ *of the Baſe* B C.

The Queſtion is; *How much is the whole line of the Baſe* B C?



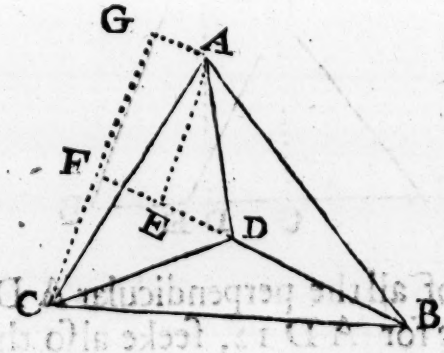
DRaw from the point C a line parallel to the ſide A B, as C E, then extend the line A D till it doth cut the ſaid line C E in the point E, then is the little Triangle C D E of one proportion with the Tri-

Triangle A B D, and then is B D to D C as 2 is to 1; and also A D to D E: therefore say, If 2 gives 1, what gives 13 for A B? and there comes $6\frac{1}{2}$ for C E: also what gives $\sqrt{162\frac{7}{9}}$ for A D? coms $\sqrt{40\frac{25}{36}}$ for D E. Then adde $\sqrt{40\frac{25}{36}}$ to $\sqrt{162\frac{7}{9}}$ comes together $\sqrt{366\frac{1}{4}}$ for A E: Then seek E F, and there shall come $\sqrt{22\frac{9768}{18600}}$, and then is F C also known, for F C is $\sqrt{19\frac{385}{14650}}$, then substract E F from E D, rests F D $\sqrt{2\frac{6754}{11385}}$, this square of F D added to the square of F C, comes for the square of D C $21\frac{7}{9}$, whose square Root is $4\frac{2}{3}$ for the line C D: then is D B $9\frac{1}{3}$, and the whole line B C 14, which was required.

Quest. 17.

There is a Triangle A B C, *of which the sides* A B *and* B C *are known,* A B 14, *and* B C 15, *and this Triangle is divided into three other Triangles, by the lines* D A, D B, *and* D C: *and if* A D *be* $\sqrt{101\frac{1}{4}}$, *and* D B $\sqrt{45\frac{1}{4}}$, *and* D C $8\frac{1}{8}$.

The Question is; *How much is the side* A C?



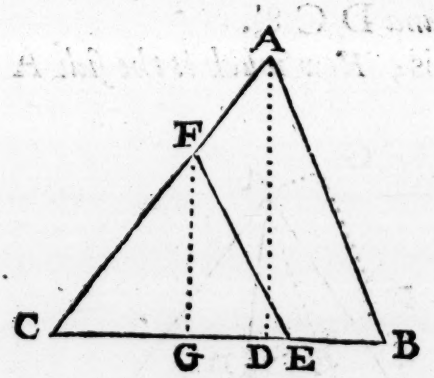
SEeke first the lines B E and A E, and there shall come for B E $\sqrt{108\frac{72}{121}}$, and for A E $\sqrt{87\frac{228}{121}}$, also seeke B F and F C, & there comes for B F $\sqrt{216\frac{228}{121}}$,

and for C F $\sqrt{18\frac{128}{181}}$: then adde F G (equall to A E) to F C comes together $\sqrt{150\frac{75}{181}}$ for the whole line G C, now take B E $\sqrt{108\frac{52}{181}}$ from B F $\sqrt{216\frac{128}{181}}$, reſts $\sqrt{8\frac{128}{181}}$ for E F or A G: and laſtly, adde the ſquare of A G, to the ſquare of G C, and there comes 169 for the ſquare of A C, whoſe ſquare Root is 13 for A C, which was required.

QUEST. 18.

There is a piece of Land in form of a Triangle, as A B C, *whoſe three ſides are known,* A B 13, B C 14, *and* A C 15, *and it is required to make a ditch in the ſame, to incloſe a part thereof, and ſhall begin on the ſide* A C, 5 *from the point* A, *that is at* F, *and ſhall end at the point* E, *that is to ſay, 4 diſtant from the point* B.

The Queſtion is; *How long ſhall this line of partition* E F *be?*



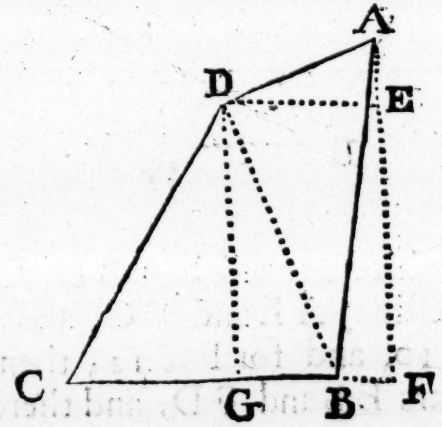
SEek firſt of all the perpendicular A D, and there ſhall come for A D 12, ſeeke alſo the part D C comes 9, then ſay, If A C 15, gives A D 12, what ſhall C F 10 give? and there comes 8 for G E. Say likewiſe, If A C 15, gives D C 9, what ſhall F C

10 give : and there comes 6 for G C, then is C G and B E together 10, which taken from B C 14, reſts 4 for E G, then adde the ſquare of E G, to the ſquare of G F, comes for the ſquare of F E 80, then is the length of the line E F √ 80, which was required.

QUEST. 19.

There is an irregular Quadrangle A B C D, *whereof the ſide* A B *is* √ $192\frac{1}{4}$, B C 13, C D 15, *and the Diagonall* D B *is* 14.

The Queſtion is; *How much is the ſide* A D *?*



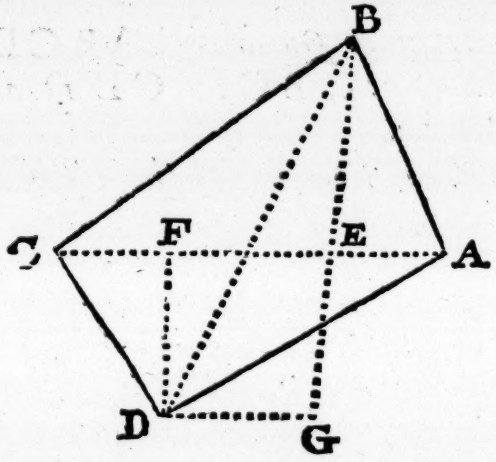
FIrſt ſeeke B G, and the perpendicular D G, and there ſhall come for D G $12\frac{12}{13}$, and for B G $5\frac{5}{13}$, and ſeek alſo A F and F B; and there comes for A F $13\frac{11}{13}$, and for F B $\frac{19}{26}$: add now $\frac{19}{26}$ to the $5\frac{5}{13}$, makes together $6\frac{3}{26}$ for F G or E D: ſubſtract now $12\frac{12}{13}$ D G, from $13\frac{11}{13}$ A F, reſts for A E $\frac{12}{13}$: this ſquare of A E, and the ſquare of E D added, comes $38\frac{1}{4}$ for the ſquare of A D, and then is the line A D √ $38\frac{1}{4}$, which was required.

QUEST.

QUEST. 20.

In this irregular Quadrangle A B C D, *the ſides are known,* A B *is* 26, B C 40, C D 20, A D 34: *alſo the Diagonall line* A C *is* 42.

The Queſtion is; *How much is the other Diagonall line* B D?



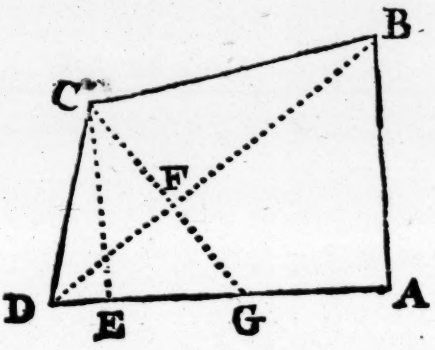
TO finde this diagonall line B D, ſeeke (by our firſt Queſtion) A E and F C, and there comes for A E 10, and for F C 12; then ſeeke both the perpendiculars E B and F D, and there ſhall come for E B 24, and for F D 16; then is B G 40. Then add A E and F C together comes 22, that taken from A C 42 reſts for F E 20, and ſo much alſo is G D: and laſtly, the ſquare of D G, added to the ſquare of G B, there comes 2000 for the ſquare of B D, and then is the diagonall line D B √2000, which was required.

QUEST.

QUEST. 21.

There is a Quadrangle as A B C D, *whereof* A *is a right angle, and the length of* A B *is* $33\frac{1}{7}$, *and* B C 39, *and* C D 25, *and* A D $44\frac{4}{7}$.

The Queſtion is; *How much is the perpendicular* C E?



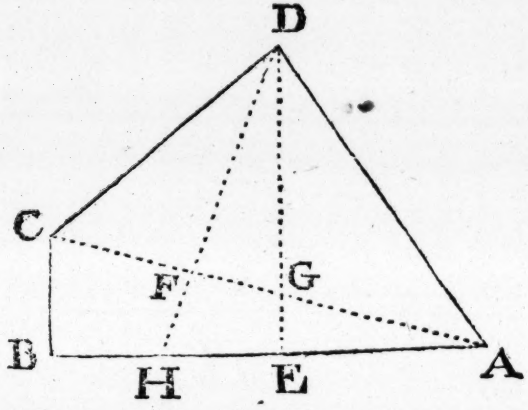
SEeke firſt D F, which may be found by reaſon that A is a right angle, and A B and A D both known, *(by the 47 of the 1 of Euclid)* and there ſhall come 56 for B D: then are the three ſides of the Triangle B C D known, and by the firſt Queſtion is known the perpendicular C F, and alſo the parts of the Baſe D F and B F, and there ſhall come 20 for D F, then is alſo F C known, and is 15.

Then ſay, If A D $44\frac{4}{7}$, gives A B $33\frac{1}{7}$, what ſhall D F 20 give? and there comes 15 for F G, this added to F C 15, there comes for G C 30. Say likewiſe, if B D 56, gives A D $44\frac{4}{7}$, what ſhall G C 30 give, and there comes 24 for E C which was required.

Quest. 22.

There is a Quadrangle A B C D, *whereof* B *is a right angle, and* A B *48, and* B C *14, and* C D *is 30, and* A D *40.*

The Queſtion is; *What is the perpendicular line* D E?



FIrſt finde A C, and there ſhall come 50 for A C; then ſeeke how much A F and F D are, and there ſhall come for A F 32, and for D F 24; Then ſay, If A B 48, give B C 14, what ſhall D F 24 give? and there ſhall come 7 for F G. Say again, If A B 48, gives A C 50, what ſhall D F 24 give? & there comes 25 for D G, take G F 7, from A F 32, reſts for A G 25: Then ſay again, If A C 50, gives B C 14, what gives A G 25? and there comes for G E 7, which added to D G 25, there comes 32 for the perpendicular D E, which was required.

The

The ſame another way.

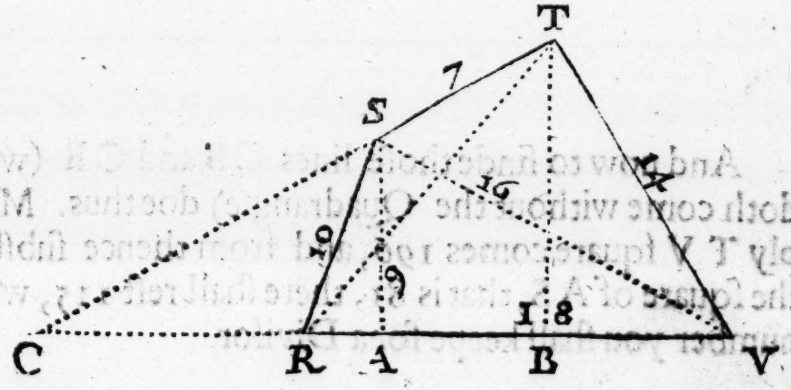
HAving found F D and F A, then ſay, If A B 48, give B C 14, what gives A F 32? and there ſhall come $9\frac{1}{3}$, this added to F D 24, comes together $33\frac{1}{3}$ for D H: then ſay, If A C 56, gives A B 48, what gives B H $33\frac{1}{3}$? and there ſhall come for D E 32 as before.

Theſe lines may be found diverſe other wayes, as may be gathered by many of our Queſtions.

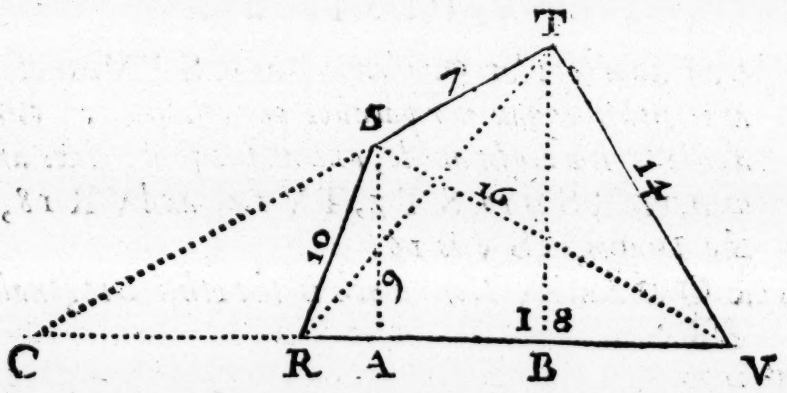
QUEST. 23.

There is an irregular Quadrangle as R S T V, *whoſe ſides are neither equall nor parallel one to another, only the angle* T *is a right angle, and all the foure ſides are given, for* R S *is* 10, S T 7, T V 14, *and* V R 18, *alſo the Diagonall* S V *is* 16.

The Queſtion is; *How much is the other Diagonall line* T R?



TO know this, draw the perpendicular lines A S and B T, then in the Triangle R S T are all the three ſides known: Multiply the two ſides including the right angle together, and divide the product by the third ſide or baſe, the Quotient is 9 for the perpendicular A S, then note, that if the lines T S and V R be prolonged forth, they will concur together in the point C, and ſo make the right angled Triangle T C V.



And now to finde thoſe lines C S and C R (which doth come without the Quadrangle) doe thus. Multiply T V ſquare comes 196, and from thence ſubſtract the ſquare of A S, that is 81, there ſhall reſt 115, which number you ſhall keepe for a Diviſor.

Then

Then multiply S T 7, by A S 9, and the product is 63: Multiply alſo A V $13\frac{3}{5}$ (before found) by T V 14, and there comes $190\frac{2}{5}$, adde theſe two together, 63 and $190\frac{2}{5}$, and there comes $253\frac{2}{5}$, which ſumme again multiplyed by 14 for T V, comes $3552\frac{8}{5}$, which product being divided by 115, the Diviſor kept, the Quotient is $30\frac{20}{23}$ for the whole line C V.

Multiply alſo the former ſum $253\frac{2}{5}$ by 9, and there comes 2283, which alſo divided by 115 (the former Deviſor kept) the Quotient will be $19\frac{98}{115}$ for C S, to this adde 7 for S T, and there comes $26\frac{98}{115}$.

Now are all the three ſides of the Triangle T C V known; for C V is $30\frac{20}{23}$, C T is $26\frac{98}{115}$, and T V is 14.

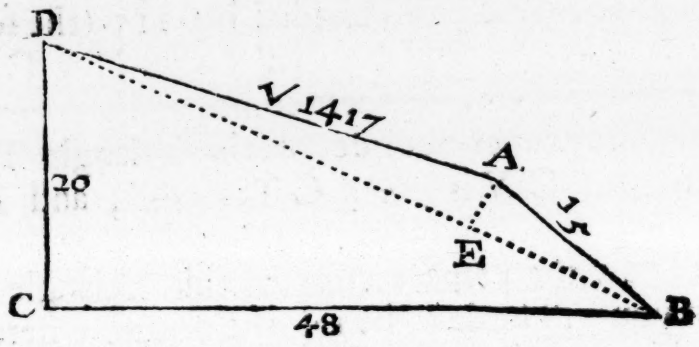
Get now the perpendicular of this Triangle B T, which you will finde to be 12, the ſquare whereof is 144, this 144 taken from the ſquare of $26\frac{98}{115}$, there reſts 577, whoſe ſquare Root is 24 for B C.

From B C 24, take C R $12\frac{20}{23}$, and there reſts $11\frac{3}{23}$ for B R.

Then adde the ſquare of T B 144, to the ſquare of R B 121, and there comes 265, whoſe ſquare root is $16\frac{3}{11}$ for the diagonall R T, which was the line required to be found.

QUEST. 24.

There is a four-angled piece of Land, as A B C D, *containing 576 Perches, whereof the angle* C *is a right angle, and the sides* A B *is 15,* B C *48, and* C D *20.*
The Question is; *How much is* A D?



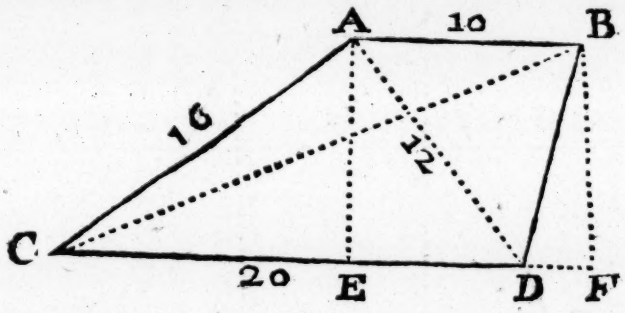
TO finde the side A D, seeke first the content of the Triangle B C D, which is 480, which taken from 576, there rests 96, for the content of the Triangle A D B, and seeing that C is a right angle, the side or Diagonall line B D may be known *by the 47 of the 1 of Euclid*, which is 52, whose halfe is 26, by which divide 96, and there comes $3\frac{9}{13}$ for the perpendicular A E: Now A E being known then is B E also known, and is $14\frac{7}{13}$, which taken from 52 the whole line B D, there rests for the part E D $37\frac{6}{13}$: then the square of E D, added to the square of E A the product is 1417, the square root whereof is $35\frac{28}{71}$ for the line A D, which was required.

QUEST.

QUEST. 25.

There is a Quadrangle A B C D *of which the upper and nether ſides are parallel, but unequall, the angles are alſo unequall: In this Quadrangle are drawn two Diagonals which do croſſe one another, as* A D *and* B C, *whereof the ſhorter Diagonall Line* A D *is known, and is* 12.

The Queſtion is; *How much is the other Diagonall line* B C?



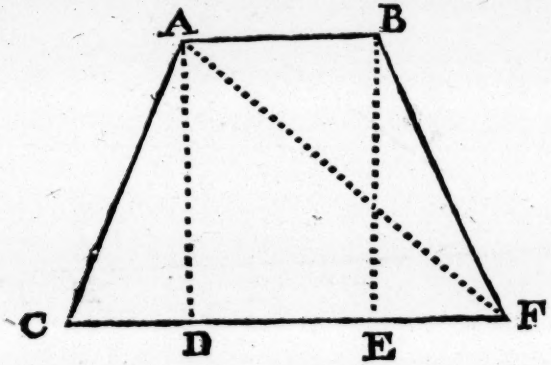
HEre note that we have two known Triangles, as A C D, and A D B, let fall a perpendicular from the point A upon the Line C D, as A E, the length of which may be known by the former inſtructions, and there ſhall come $9\frac{3}{5}$ for the perpendicular A E, and ſo much alſo is B F: then ſeeke D F, ſubſtract the ſquare of B F, from the ſquare of B D, reſts $8\frac{4}{25}$, whoſe ſquare Root is $\sqrt{8\frac{4}{25}}$, then is the whole line C F $2\frac{4}{25}$, which multiplyed in himſelfe comes $\frac{2499}{25}$, which added to the ſquare of B F $\frac{2304}{25}$, comes 612, whoſe ſquare root is $24\frac{3}{5}$ the length of B C, which was required.

QUEST.

Quest. 26.

There is a Quadrangle A B C F, *whose upper line* A B *is parallel to the Base* C F, *and also both the angles* A *and* B *are equall, and also the angles at the Base, as* C *and* F: *Now* A B *is* 8, *and* A C *is* 13, *equall to* B F, *and* C F *is* 18.

The Question is; *How much is the perpendicular* A D, *and also how much is the line* A F?

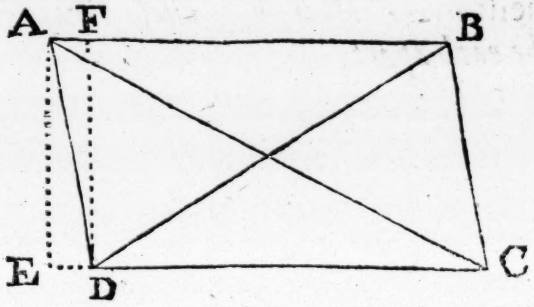


MAke D E equall to A B 8, then must C D and E F (being equall) together make 10, and each alone 5, whose square is 25, which being taken from the square of A C 169 rests 144 for the square of A D, whose square Root is 12 for A D. And to know the line A F, adde the square of A D 144, to the square of D F 169, there comes 313 for the square of A F, whose square Root is $17\frac{24}{25}$, for the line A F, which was required.

QUEST. 27.

There is an irregular Quadrangle whoſe ſides oppoſite are equall and parallel one to another, that is A B *and* D C, *each being* 17, *and* A D *and* B C *each* 10; *and the Diagonall* A C *is* 21.

The Queſtion is; *How much is the other Diagonall* BD?

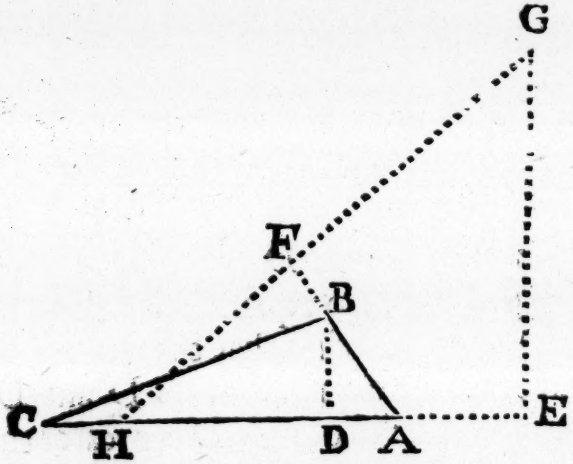


IN this given figure the Diagonall A C divideth the Quadrangle into two equall Triangles, as A B C, and A D C, and ſeeing that the ſquare of A C is greater then both the Squares of A D and D C together, it followeth that the angle D is greater then a right angle, wherefore from the ſharp angle A let a perpendicular be drawn as A E, the ſame falleth without the Triangle, ſeeke therefore how much is A F (equall to E D) and there ſhall come $1\frac{9}{17}$ for E D, whoſe ſquare taken from the ſquare of A D, reſts $\frac{28224}{289}$, then is the line A E $\frac{168}{17}$, or $9\frac{15}{17}$, Make now A F equall to D E, and draw D F perpendicular upon A B equall to A E, and then is A F D E a parallelogram, or of parallel lines. Subſtract now A F $1\frac{9}{17}$ from A B 17, reſts $15\frac{8}{17}$, whoſe ſquare added to the ſquare of the former D F, comes 337, whoſe ſquare Root is $18\frac{13}{17}$, for the Diagonall BD.

Quest. 28.

There is a Triangle A B C, *whereof the ſides are known,* A B 10, B C √ 640, *and* A C 30: *and if the line* A B *be prolonged* 5 *longer, that is to* F, *and* C A 11 *longer to* E, *and from* E *a line drawn to* G, *as* E G, *and from* F *a line drawn to* G, *as* F G, *ſo that* A F G *and* A E G *be both right angles.*

The Queſtion is; *How long theſe lines* E G *and* F G *ſhall be each apart,*



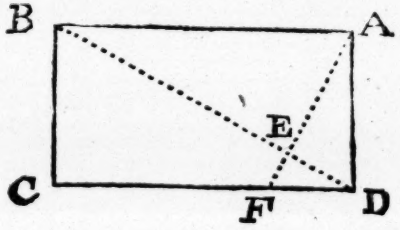
SEeke firſt A D and D B, and there ſhall come for A D 6, and for B D 8, produce G F to H, then is the Triangle A FH of like proportion to the Triangle G E H, and alſo like to the Triangle A B D: therefore ſay, If A D 6, gives A B 10, what ſhall A F 15 give? and there ſhall come 25 for A H, to this adde 11 for A E, comes together 36 for E H: ſay again, If A D 6, give D B 8, what gives A F 15? and there comes 20 for F H: and becauſe we muſt find E G, ſay, If B D 8, gives A D 6, what gives E H 36? and there ſhall

ſhall come 27 for E G: alſo, If B D 8, gives A B 10, what gives E H 36? and there comes for G H 45, from hence ſubſtract F H 20, and there reſts for F G 25, and ſo are both thoſe lines found F G 25, and E G 27.

QUEST. 29.

There is a right angled Quadrangle A B C D, *in which is meaſured upon the Diagonall* B D *from* B *to* E, *where the perpendicular falling from the angle* A, *doth cut the ſaid Diagonal at right angles in the point* E, *as* A F, *ſo that* E B *is 16, and* E F *is 2.*

The Queſtion is; *What is the Content or Area of that Quadrangle?*

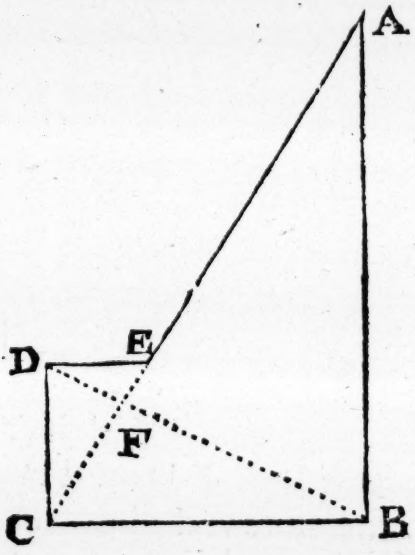


THe foure angles being right angles, therefore E F is to E D, as E D to E A, and as E A to E B: therefore ſeeke how much E D and E A are, thus, Set E F 2, and E B 16 in their leaſt proportion, which is 1 and 8, and becauſe E B is the third number from E D, therefore extract the Cubick Root from 8 (in his leaſt proportion) comes 2 for A E, and E D is 1 in his leaſt proportion (and being multiplyed ſquare ſhould bring forth the leaſt proportion for A E, & Cubically multiplyed brings forth the leaſt for E B) & then ſeeing that E F was diminiſhed by 2, therefore increaſe the Root found by 2, and there comes 4 for E D. Then is E A 8, and the whole content of the Quadrangle 128.

QUEST. 30.

There is a five-angled piece of Land, as A B C D E, *whereof the angles* B C *and* D *are right angles, and if the line* A E *be prolonged forth, it comes in the angle* C, *alſo if* B D *be drawn, then is it found that the line* A C *doth cut the line* B D *at right angles in* F.

The Queſtion is; *If* A E *be 30, and* E F *2, how much is then the content or Area of the aforeſaid five-angled Figure.*



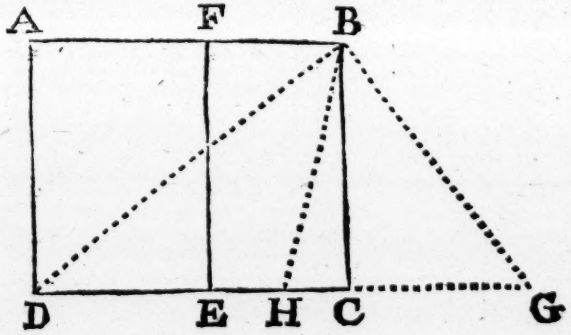
ADde E F 2 to A E 30, comes 32, this 32 multiplyed by 2 comes 64, from hence extract the ſquare Root, comes 8 for F C (becauſe F C is a mean proportionall betweene F E and E A) this 8 multiplyed again by 2 comes 16, whoſe ſquare Root is 4 for F D: and F D is a meane proportionall between F E and F C. And now ſeeing that F C and F D are both known, by the ſame means may B F be known, or ſay,

ſay, If F E 2, gives F D 4, what gives F C 8? and there comes 16 for F B: then multiply 8 the halfe of 16, by 40, the whole line A C, and there comes 320 for the Triangle A B C. And again, The halfe of F D, that is 2, multiplyed by E C 10, comes 20 for the Triangle E C D, which added to 320, there comes for the Area of the whole five-angled figure A B C D E 340, which was required.

Quest. 31.

There is a right angled Quadrangle, whereof the breadth A D *is* 24, *and if we cut off from the ſame the piece* F B C E, *parallel to* A D *or* B C, *containing* 336, *then is the reſt as* A F E D *of like proportion to the whole Quadrangle* A B C D.

The Queſtion is; *How long is then this aforeſaid Quadrangle, as* A B *or* D C?



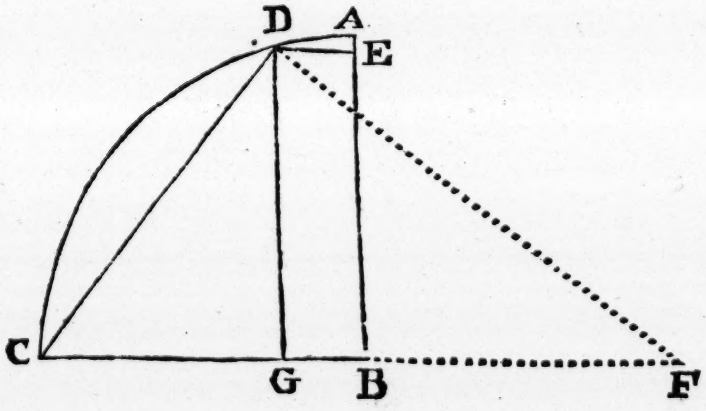
Divide 336 by 24, comes 14 for E C, then make G C equall to E D, and then is the midſt between E C, namely H, alſo the midſt between D G, and then is H C 7 (becauſe E C is found to be 14) now as C G is to C B, ſo is C B to C D, therfore is D B G a right angle. There-

Therefore ſeeke H B, by multiplying H C in himſelfe ſquare comes 49, and B C alſo (equall to A D) 24 multiplyed ſquare comes 576, which two ſquares added, comes 625 for the ſquare of B H, whoſe ſquare root is 25, and ſo much is D H, unto which adde 7 for H C, coms 32, for the whole line D C or A B, which was required.

QUEST. 32.

There is a Quadrant or fourth part of a Circle, as A B C D, *in which is made a Quadrangle* E B C D, *in ſuch ſort that* E D *is parallel to* B C, *and that* E D *and* B C *together is juſt as long as* C D, *now* C D *is* 10--- $\sqrt{20}$.

The Queſtion is; *How much ſhall be the ſides of the Quadrant* A B *and* B C?



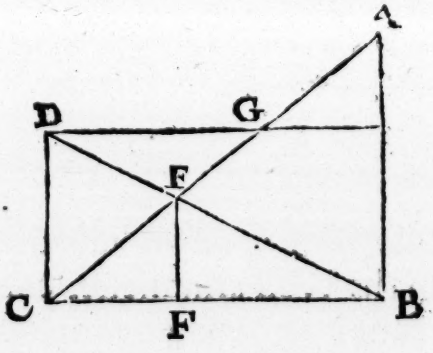
BEcauſe E D and B C together are as much as C D alone, therefore is F G equall to C D, ſeeing that F B is equall to B C, and B is the centre of the Quadrant, and ſo is F D C a right angled Triangle in the

the angle D, in which C G is to D G, as G D to G F: and for that F G is equall to C D, therefore is C G to G D, as G D is to D C, and alſo, C G is to D C or F G, as D C to F C: hereby it appeareth, that the line F C is divided into two ſuch parts (that is F G and G C) as that C G is to F G, as F G is to the whole line F C. And now that F G (being equall to D C) is known to be 10---√20, we ſeeke then G C, and there ſhall come for G C √180---10, and for F C the whole line comes √180, whoſe halfe is √20 for B C which was required.

QUEST. 33.

A Carpenter hath made a Gate in form like this figure, whereof A B *is 9 foot,* B C *is 10 foot, and* C D *is 6 foot.*

The Queſtion is; *How much then is* A E, E C, E B *and* E D, *where the croſſe woods* A C *and* B D *doe cut one another, and alſo how much* E F *and* D G *are each apart.*



ADde 9 to 6 comes 15, then ſay, If 15 gives 10, what gives 6? and there comes for F C 4, or, If 15 gives 10, what gives 9? and there comes for B F 6: Then is the line B C ſo divided into two parts, that

that B F is to A B as C F is to C D; then ſay, If B C 10, gives A B 9, what gives F C 4? and there comes $3\frac{3}{5}$ for F E, and *by the 47 of the 1 of Euclid*, then is E C and E B known alſo, E C $\sqrt{28\frac{24}{25}}$, and E B $\sqrt{48\frac{24}{25}}$, and then is E D $\sqrt{21\frac{19}{25}}$ and A E $\sqrt{65\frac{4}{25}}$, and as E C to E B, ſo E G to E D, and, as E B to E D, ſo B C to D G, ſo that D G is alſo made known: then ſay, If E B $\sqrt{48\frac{24}{25}}$, gives E D $\sqrt{21\frac{19}{25}}$, what gives B G 10? and there coms $\sqrt{44\frac{4}{5}}$ for D G, and ſo is this Queſtion ſolved.

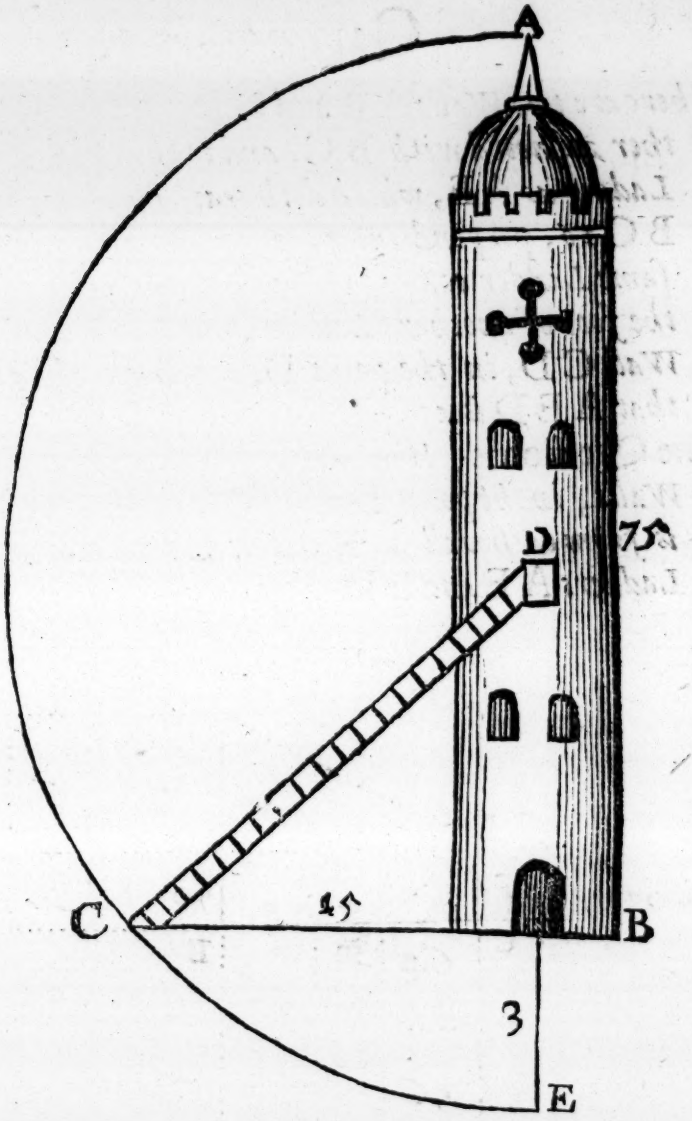
Quest. 34.

There is a Towre whoſe height A B *is known to be 75, and againſt the ſame is a Ladder ſet, ſo that the foot of the Ladder ſtandeth out from the bottome of the Towre 15, to come to a Window, If then from the Window to the top of the Towre be equall to the length of the Ladder,*

The Queſtion is; *How much is beneath the top of the Ladder to the ground, and alſo how long is the ſaid Ladder?*

Multiply B C 15, the diſtance of the Ladder from ths Wall in himſelfe, comes 225, divide the ſame by 75 comes 3 for B E, and the reaſon of this is, that A B is to B C, as B C is to B E; and alſo becauſe D A is equall to D C the length of the Ladder, and now B E being known, adde B E 3, to B A 75, and there comes 78 for A E, the halfe whereof

is

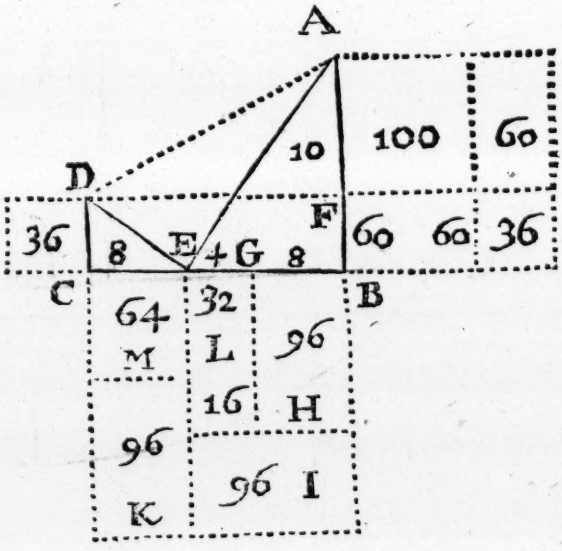


is 39 for A D or D C the length of the Ladder: And if we ſubſtract B E 3, from D E 39, there ſhall remain 36 for BD, the height of the window from the ground, which was required.

Quest. 35.

There are two Walls A B *and* C D, *distant one from another* 20, *noted with* B C, *and to the Wall* A B *is set a Ladder as* A E, *which doth reach up above the ground* B C 16, *perpendicular, as to* A, *and the foot of the same Ladder comes to* E, *and at the same point* E, *coms the foot of another Ladder, and doth reach upon the Wall* C D, *in the point* D, 6, *above the ground, so that* A E D *is a right angle.*

The Question is; *How much those ladders stand from the Walls, in the point* E *or, how much these at the point* E *is from each wall at* B *and at* C, *also how long these Ladders* A E *and* E D *are.*

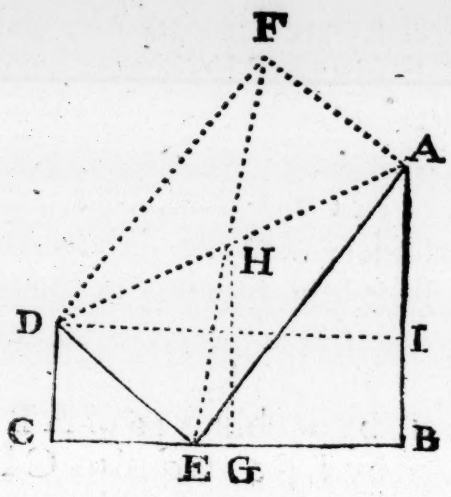


THe square of A B and of B C, with the squares of E C and of C D, those 4 squares together are equal, to the square of A D alone, (*by the 47.*

of

of the 1 of Euclide) alſo the ſquares of A F and of B C, together are equal to the aforeſaid ſquare of A D alone, it followeth then, that the 2 ſquares laſt named, A F and B C, together are equal to the 4 aforeſaid ſquares, of A B, B E, E C and C D, if we then take the ſquare of A F, from the ſquare of A B, then muſt the remainder of A F, with the 3 other ſquares, as B E, E C and C D, together be equal to the ſquare of B C, and the aforeſaid remainder, with the litle ſquare of C D, is equal to two of the long ſquares, as H and I; and ſeeing that in the ſquare of B C, there is contained 4 of thoſe long ſquares, as H I K M L, ſo can the contents of them be well known, by doubling 16 of A B comes 32, and that multiplied by B F 6, (equal to C D) comes 192 for the 2 pieces H and I, this doubled comes 384 for the 4 long ſquares H I K M L, now the ſquare of B C is 400, from which ſubſtract 384 remaines 16 for the ſquare of G E, whoſe ſquare Root is 4 for G E, this 4 taken from B C 20 reſt 16, whoſe halfe is 8 for B G or E C, then is B E 12: the length of the ladders are then known, *(by the 47 of the 1 of Euclid.)* And there comes for A E 20 and for E D 10 which was required.

The ſame another way.



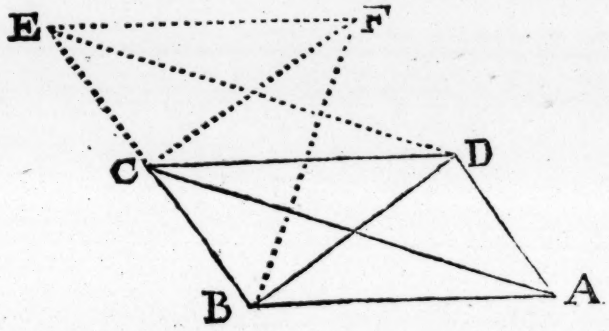
LEt the Walls be A B and C D, as before, and A E one Ladder, and E D the other, and B C the diſtance of the Walls, and A E D a right angle: then make the Quadrangle A E D F, whoſe Diagonall line is A D or E F, whoſe midſt is H: then ſubſtract C D 6, from A B 16, and there will reſt for A I 10, and I D is 20, equal to the diſtance of the Wals B and C, wherefore *(by the 47 of the 1 of Euclid)* A D is known, and is √500, the halfe whereof is √125, and ſo much is H E. Then adde D C 6, to A B 16, and there comes 22, the halfe is 11 for H G (becauſe H is the middle point of A D, and G the middle point of B C) and ſeeing that H G and H E are both known, therefore is G E alſo known, and there coms for G E 2, which taken from C G 10, reſts for E C 8, or added to B G coms for B E 12 as before.

QUEST.

QUEST. 36.

There is a depreſſed Quadrangle, as A B C D, *of which the ſides* A B *or* C D *are each of them* √ *106, and the Diagonall* A C *is* 12, *and* B D 10.

The Queſtion is; *If the ſide* A D *be equall to* B C, *how much then is* A D *or* B C.



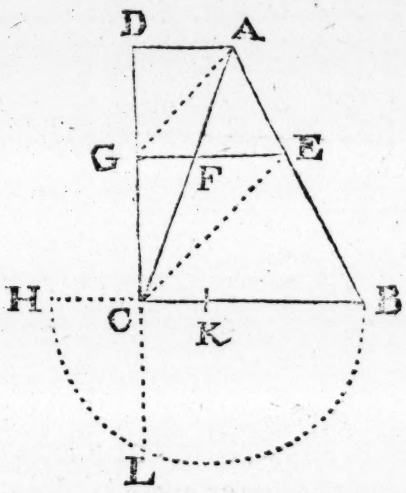
TO finde the ſame, note that the ſquare of B D and E D, are alwayes as much as the ſquare of B F and F C together are, If then the line D C falleth perpendicular upon the Baſe B E in the point C, being upon the halfe of B E, as in this, the line F C being alſo √ 106 equall to C D, then is B F and E F alſo equall: This then being ſo, adde both the ſquares of B D and D E together, that is of 12 and of 10, and there coms 244, whoſe halfe is 122 for one of the ſquares of B D or F E, from hence take the ſquare of F C, being 106, there ſhall reſt 16 for the ſquare of B C or of C E, then is B C or A D 4, which was required.

This Queſtion may be reſolved in manner of the ninth Queſtion.

Quest. 37.

There is a Quadrangle A B C D, *whereof* A D *is parallel to* B C, *and* A B *is* 25, B C 18, *and* C D $21\frac{2}{3}$, *and* A D *is* 8, *in which is drawn a parallel line* E G, *in such sort that the Triangle* A B C *is equall to the Quadrangle* E G C B.

The Question is; *How long is the parallel line* E G, *and also how long are the lines* A E *and* D G?



TO doe this, Make C H equall to A D, so that B H be as much as B C and A D together, of this line B H, the middle point is K, make then upon the centre K a halfe Circle B L H, and from C draw a perpendicular C L, then is C L a mean proportionall between B C and C H; therefore multiply C H 8, by B C 18, comes 144 for the square of C L, whose square root is 12 for C L, and so much also is E G a mean proportionall between A D and B C, and that is thus

thus proved; for as much as the Triangle A B C is given, equall to the Quadrangle B E G C, let us draw the line E G, then doth the ſame cut off the Triangle C G E, which muſt be equall to the Triangle C A E: therefore we draw alſo the line A G, and ſo now two Triangles doe ſtand upon one Baſe E C, whoſe contents are equall.

It now followeth *(by the 37 of the 1 of Euclid)* that then the line A G is parallel to E C, and A G is then the Diagonall line of the Quadrangle E G C B; and furthermore, becauſe A D is parallel to E G, and A G to E C, therefore the Triangles A G E, and E C B, are of like proportion: Now are both the Triangles that make the Quadrangle A D G E, of like proportion to thoſe that make the Quadrangle E C G B; from whence it followeth that the Quadrangle A D G E, is of like proportion to the Quadrangle E G C B, then is B C to E G, as E G is to A D, wherefore E G is the mean proportionall between B C and A D, which was to be proved.

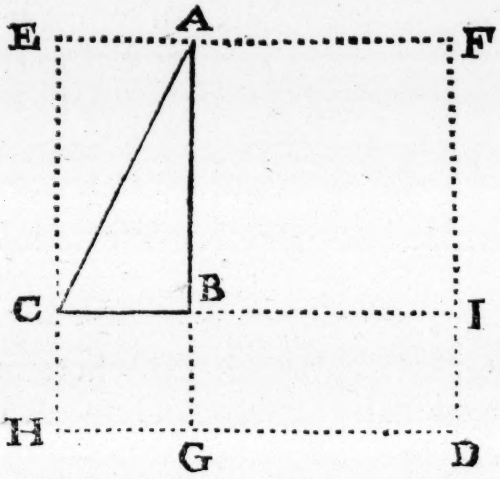
For as much then as B C is to C G, as E G to G D; or as B C to B E ſo E G to E A: hereby is made known both A E and D G, ſaying, E G 12, and B C 18, is together 30, theſe are to divide A B 25, what ſhall E G 12, and what B C 18? and there ſhall come of 12 for A E 10, and of 18 ſhall come for E B 15: In the ſame manner ſeeke G D and G C, and there ſhall come for G D $8\frac{1}{2}$ and for G C 13, which was required.

QUEST.

QUEST. 38.

There is a Triangle A B C, *right angled in* B, *whereof is known, that the ſides* A B *and* B C *together are* 17, *and* A C *alone is* 13.

The Queſtion is; *How much* A B *and* B C *are each apart?*



OF the lines AB and BC together, make a Square, as E F D H, the ſide thereof then is 17, and the content of the ſquare is 289. Alſo multiply A C in it ſelfe ſquare, comes 169 for the contents of both the Squares of A B and B C *(by the 47 of the 1 of Euclid,)* This 169 ſubſtracted from 289 reſts 120. Then the ſquare of both the lines A B and B C together is greater then the ſquare of A C alone, which ſquare of A C is equall to the ſquares of A B and B C apart. The halfe of this 120 is 60, for one of the parts B E,

B E or B D, which peeces are double to the Triangle A B C, and are the mean proportionall between the ſquare of A B and the ſquare of B C: Then ſeeke by the fourth Queſtion, how much A B and B C are each apart, and there ſhall come 25 for the ſquare of B C, and 144 for the ſquare of B A: So the line A B is 12. and B C is 5, which was required.

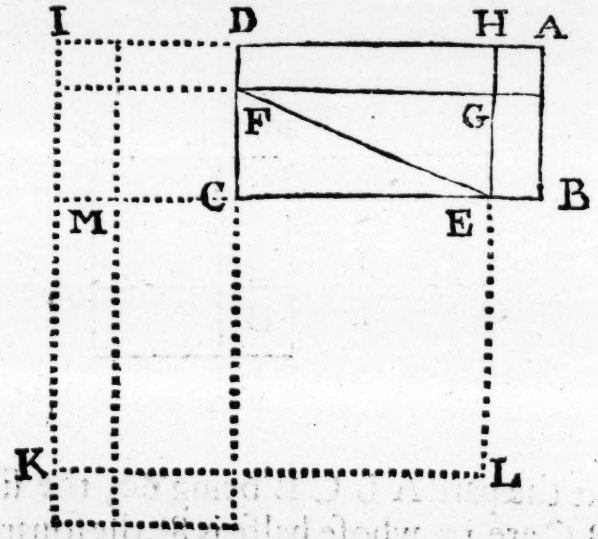
Another way.

OR take the part A B C E being 60, the ſides A B and B C are 17, whoſe halfe is $8\frac{1}{2}$, the ſquare whereof is $72\frac{1}{4}$, from which take 60, and there reſts $12\frac{1}{4}$ for the ſquare G, whoſe ſquare Root is $3\frac{1}{2}$ for the ſide, or for F H, that taken from E F $8\frac{1}{2}$, reſts 5 for E H or E A, or B C, and the ſame $3\frac{1}{2}$ added to $8\frac{1}{2}$, there comes 12 for A B required.

QUEST. 39.

There is a right angled Quadrangle, as A B C D, *whose Area or content is 98, wherein is made a line of division, as* E F, *whose length is 13, so that* D F *is 2, equall to* B E.

The Question is; *How much is the length and breadth thereof, that is, How long is* A D *and* A B?



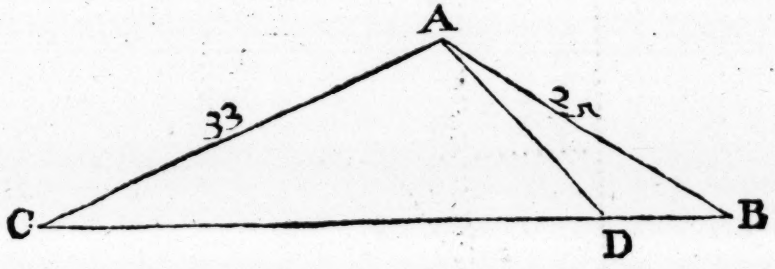
MUltiply 13 in himselfe square comes 169, to which adde twice 98, that is, 196, comes together 365, multiply also 2 for B E in himself, coms 4, that taken from 365, rests 361 for the Square H I K L, whose square Root is 19 for I H, or the side of that Square, from hence take 2, rests 17 for E C M, or for E C and C F together, and now to finde E C and C F each a part: that may be done for as much as E F is known, as is shewed in our last Question, & there shall

ſhall come 5 for CF, and 12 for EC, then is the breadth A B 7, and the length B C 14, which was required.

QUEST. 40.

There is a Triangle A B C, *whereof* A B *is* 25, *and* A C *is* 33, *in which is drawn a line* A D, *from the angle* A, *cutting the Base* B C *in* D, *ſo that* B D *is to* D C, *as* 1 *to* $4\frac{19}{176}$, *alſo* B D *is to* A D, *as* 1 *to* $2\frac{1}{16}$.

The Queſtion is; *How much is the Baſe* B C, *and alſo how much is the line* A D, *and likewiſe how much are the parts of the Baſe* B D *and* D C, *each apart?*

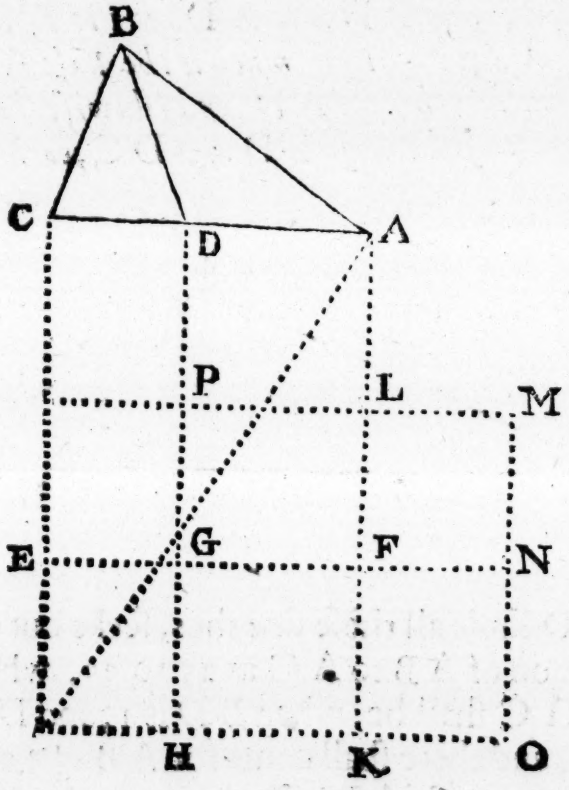


TO finde all theſe doe thus, ſeeke out the proportion of A B to A C as 25 to 33, what A B and A C ſhall be, when B D is 1, and D C $4\frac{19}{176}$, and A B $2\frac{1}{16}$, and there ſhall come for A B $2\frac{179}{238}$, and for A C $3\frac{807}{1064}$, then ſay, If A B $2\frac{179}{208}$ gives 25, what ſhall 1 give? and there coms for B D $8\frac{88}{119}$, and again, what ſhall $4\frac{19}{176}$ give? and there coms $43\frac{21}{119}$ for D C, and again once more, what ſhall $2\frac{1}{16}$ give? and there ſhall come $18\frac{1}{119}$, for A D, and the whole Baſe B C is then $52\frac{1}{119}$, which was required.

Quest. 41.

There is a Triangle A B C, *whereof the sides* A B *and* A C *are each* 10, *and the angle* A *is halfe as great as the angle* B *or* C.

The Question is; *How much is the side* B C?



FOr as much as the angles B and C are each of them twice as great as the angle A: Then draw out of one of the Angles (either B or C) a right line, as the line B D, so that the angle B is divided into two equall parts by the line B D, cutting through the line

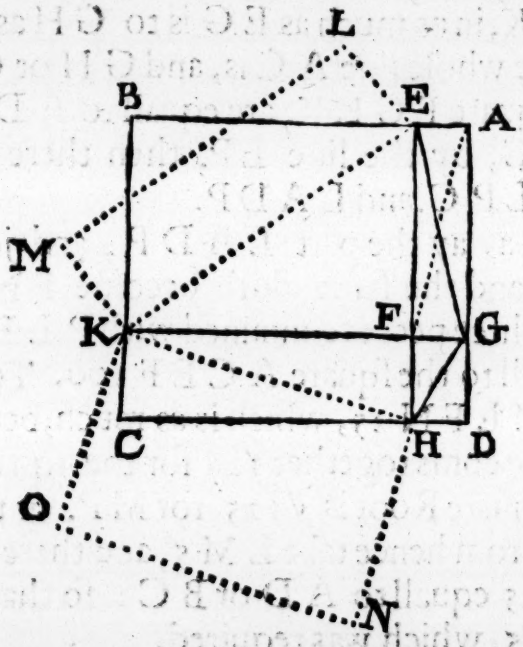
line A C in the point D; then is A D equall to D B, becauſe the angle D B A is equall to the angle D A B, alſo becauſe that the angle B D C is equall to B C D, then is the little Triangle B D C of one proportion to the great Triangle A B C, wherefore then doth D C ſtand to D A, as D A to the whole line A C: make then of A C a ſquare as here is A C E F, whoſe content is then 100, and if we ſubſtract D G H, then ſhall the right angle figure D G E C be equall to the ſquare F G H K, in as much as E G is to G H as A D to A F, ſo as the whole line A C is, and G H or G F, the ſides of the ſquare F G H K, are equall to A D: divide then F A D G, by the line L P, then there comes equall parts F L P G, and L A D P.

Take away the part L A D P, and joyne the ſame to K F, and the ſame doth become F N O K. Then are the three peeces contained with P L F N O K H P, yet equall to the ſquare A C E F 100. To this adde the ſquare M L F N 25, which is as much, becauſe L F is 5, and there coms together 125 for the ſquare M P H O, whoſe ſquare Root is √125 for M P, or the ſide of the ſquare, from hence take L M 5, and there reſts for L P √125---5 equall to A D or B C: ſo that B C is then √125---5, which was required.

Quest. 42.

There is a Quadrangle as A B C D, *in which is drawn two right lines, the one* E H *parallel to* A D, *the other* G K *parallel to* C D, *which cut one another in the point* F: *so that* F G *is to* F H, *as* F E *to* F K. *Now if* E G *be* √90, G H 5, H K √106, *and* E K 15.

The Question is, *How much is the content of the Quadrangle* ABCD: *and also the length and breadth thereof?*



UPon the line E K make a right angle figure E K M L of like proportion, as A E F G, or F K C H, that is, the Quadrangle E K M L equall to both the Quadrangles A E F G, and F K C H together *(by the 31 of the 6 of Euclid)* like as the square of E K is alone as much as both the Squares of E F and F K together, and seeing now that E K is known 15, then is E L and K M also known, and is equall to G H 5, for as much as the length of E F and F K are given, therefore

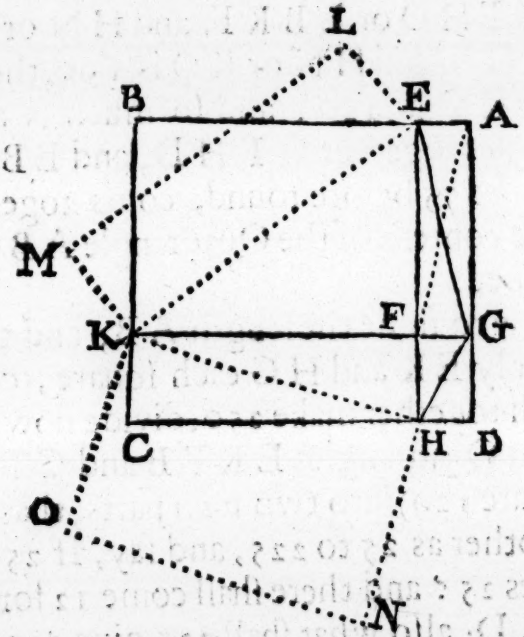
therefore doth the lines F G and F H give the breadth G H in the ſame proportion, and becauſe E K M L is of one and the ſame proportion with A E F G, or F K C H: then muſt E L be equall to G H, and therefore is 5, and then is the content L M K E 75, and ſo much is alſo the two Quadrangles A E F G, and F K C H together: And after the ſame manner alſo may be known the other two Quadrangles G F H D, or E B K F both together, becauſe H K O N is of like proportion to G F H D or E B K F, and H N or K O is made equall to G E, and H N or K O is √90, then is the content of H K O N 120, and ſo much is alſo both the Quadrangles together G F H D, and E B K F, which added to the 75 before found, coms together 195 for the whole content of the Quadrangle A B C D, which was required.

And now to finde the length A B, and the breadth B C, multiply E K and H G each ſquare, coms 225 and 25, which together make 250; divide now the contents of the two Quadrangles E K F B and G F H D (which together are 120) into two ſuch parts, that the one part is to the other as 25 to 225, and ſay, If 250 gives 120, what gives 25? and there ſhall come 12 for the content of G F A D; alſo what ſhall 225 give? and there ſhall come 108 for E B K F; or the ſquare of G E 90, and H K 160, is alſo together 250; ſay then, If 250 gives 75, the contents of both the Quadrangles A E F G, and E K C H, what ſhall 90 give? and there coms 27 for the content of A E F G; and what ſhall 160 give, coms 48 for F K C H. Let now F G be 1, then is F E 3, and divide 27 by 3 the Quotient is 9, whoſe ſquare Root is 3 for F G, then is F H 4, and F K 12, F E 9, & the length A B 15, and the breadth B C 13, which was required.

Another

Another way.

Suppose that F G is 1, then is F E 3, and F H $1\frac{1}{3}$, and F K 4, the breadth then is $4\frac{1}{3}$, and the length is 5, Multiply E H $4\frac{1}{3}$, by G K 5, and there coms $21\frac{2}{3}$, Also multiply each in himselfe square, and of $4\frac{1}{3}$ coms $18\frac{7}{9}$, and for G K 25. Then say, If $21\frac{2}{3}$ gives 195



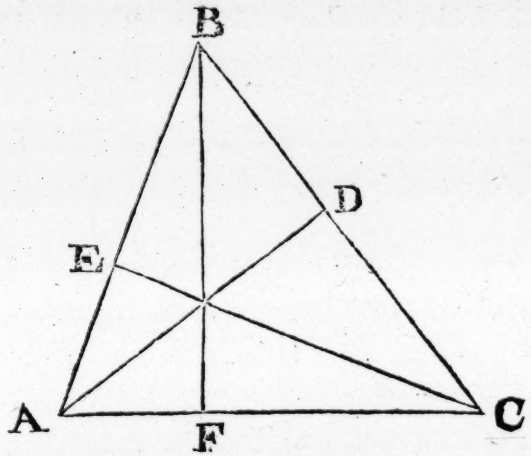
the whole content of A B C D, what gives $18\frac{7}{9}$, and what gives 25: and there shall come of $18\frac{7}{9}$ the square of the breadth E H 169, and of 25, coms for the square of the length G K 225: then extract the square Root from each of those squares, and there coms 13 for E H or A D, and 15 for K G or A B, the breadth and length required.

Quest.

QUEST. 43.

In this Triangle A B C, *are all the three perpendicular lines known,* A D *56,* B F *60, and* E C $64\frac{8}{13}$.

The Queſtion is: *How much are the three ſides of the Triangle* A B, A C, *and* C B, *each apart?*

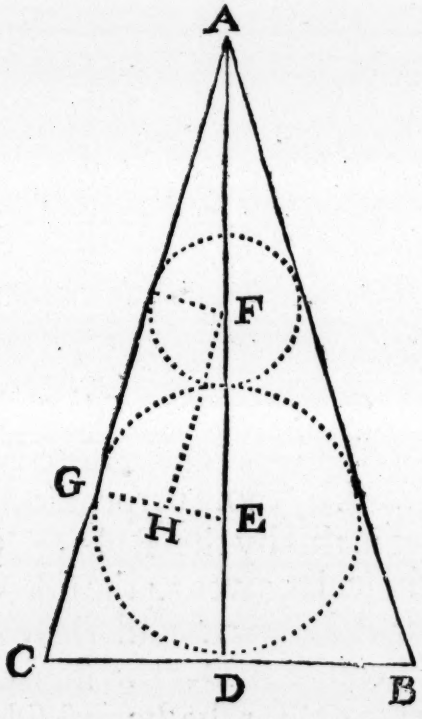


IN the firſt Queſtion is the ſide found to be 14, and the perpendicular 12, therefore by the ſame proportion work by thoſe perpendiculars given, and ſo finde the ſides in this by the Rule of Three, ſaying, If 12 the perpendicular, gives 14 the ſide, what ſhall 56 the perpendicular A D give? and there ſhall come for A B $65\frac{1}{3}$: (and becauſe A D is the ſhorteſt perpendicular, therefore is that $65\frac{1}{3}$ for the ſhorteſt ſide.) Say again, If 12 gives 14, what gives 60? and there ſhall come 70 for A C (the mean ſide.) And again, If 12 gives 14 what ſhall $64\frac{8}{13}$ give (the longeſt perpendicular?) and there ſhall come $75\frac{5}{13}$ for B C the longeſt ſide.

QUEST. 44.

There is a Triangle, as A B C, *whose two sides* A B *and* A C *are equall, in the same is set two Circles, the Diameter of the one is 8, and the Diameter of the other is 12, these Circles touch one another, and also touch the sides of the Triangle.*

The Question is; *How much the sides of this Triangle are?*



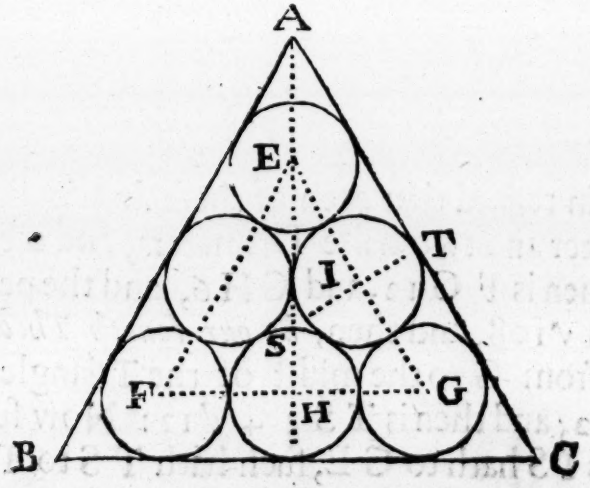
TAke the halfe Diameter of 8, from the halfe Diameter of 12, that is 4 from 6, and there rests 2 for E H, also adde both the halfe Diameters 6 and 4, and there coms 10 for E F, then say, If E H 2, gives E F 10, what gives E G 6? and there coms 30 for

for A E, to this adde 6 for E D, and there coms 36 for A D the perpendicular of the Triangle. Subſtract now the ſquare of E G, from the ſquare of A E, reſts 864 for the ſquare of A G, then is A G √864; then ſay again, If A G √864, gives A E 30, what ſhall A D 36 give? and there ſhall come √1350 for A C, and ſo much alſo is A B, and for as much as A D was known before to be 36, *by the 47 of the 1 of Euclid*, B D or D C may be known, and there ſhall come for the Baſe B C √216, and ſo are all the three ſides known, which was required.

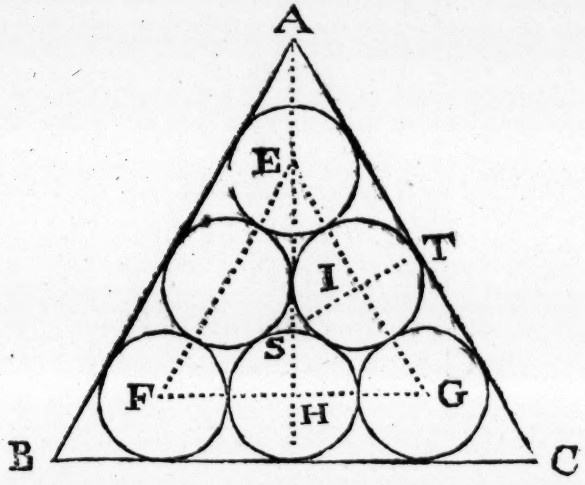
QUEST. 45.

There is an Equilaterall Triangle, as A B C, *in which is made ſix Circles of equall and like greatneſſe or quantity, which Circles are ſet ſo cloſe together as may be, and each Circles Diameter is 6.*

The Queſtion is; *How much ſhall be the ſide of this Triangle* A B C?



THe ſemidiameter of each Circle is 3, therefore draw a line through the centers of the Circles, and they ſhall make a Triangle in form like the given Triangle, for that the ſides are parallel to the given, as E F parallel to the ſide A B, and E G parallel to A C, and F G to B C, and each of theſe lines are 12.



Now in regard that each of theſe do contain one whole diameter and two halfe Diameters, and each Diameter is 6, then is F G 12, and G H 6, and the perpendicular E H is √108, and then, *by our fourth Theoreme*, is the ſpace from G to the midſt of the Triangle, namely G S, √12, and then is T S 3 + √12: Now ſuch proportion as I S hath to G E, ſuch hath T S to T A, therefore

fore ſay, If $\sqrt{12}$ of I S, gives 6 of I E, what ſhall $3+\sqrt{12}$ of T S give? and there ſhall come $6+\sqrt{27}$ for T A, which being doubled comes $12+\sqrt{108}$, for the ſide of the containing Triangle A B C, which was required.

The ſame another way.

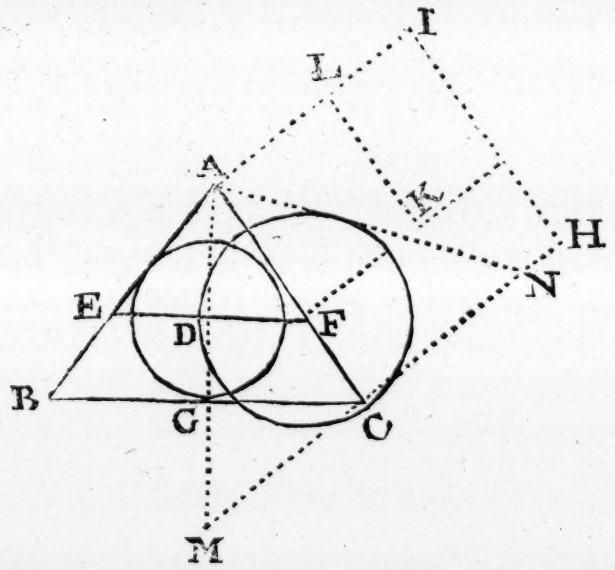
WE have ſhewed in our fourth *Theoreme*, that in all Triangles the perpendicular lines drawn from the midſt of the ſide to the oppoſite or upper angle, doth cut one another in one point, and that the part of the line from the aforeſaid point of cutting to the angle, is alwayes double to the other part of the ſame line, from the aforeſaid point to the ſide of the Triangle, wherefore ſeeke the length of the perpendicular of this Triangle (by the former inſtruction) as of E H, coms $10\frac{8}{21}$ or $\frac{218}{21}$, whereof $\frac{1}{3}$ is $\frac{73}{21}$ for S I, that is $3\frac{10}{21}$. Unto this adde 3 for I T, and there coms $6\frac{10}{21}$ for S T.

Then ſay, If $3\frac{10}{21}$ for S I, gives 6 for I E, what gives $6\frac{10}{21}$ for S T? and there ſhall come $11\frac{278}{1533}$ for A T, which being doubld, comes $22\frac{546}{1533}$ for the ſide A C, required.

Quest. 46.

There is a Triangle as A B C, *whereof two ſides* A B *and* A C *are equall, and each of them* 40, *and within this Triangle is a Circle which doth touch the three ſides of the ſame, and if a line be drawn through the centre of the ſaid Circle* D, *and parallel to the Baſe* B C, *as the line* E D F, *the ſame line doth divide the Circle into two equall parts, but in the Triangle it doth cut off the part* A E F, *which part is* $\frac{25}{64}$ *of the whole content of this Triangle* A B C.

The Queſtion is; *How long is the Baſe of this Triangle, namely, the line* B C?



MUltiply A C 40 in himſelfe, coms 1600 for the ſquare A C H I, from hence ſubſtract $\frac{25}{64}$ of the whole, coms 625 for the ſquare A F K L; becauſe the ſquare A C H I hath proportion to the Triangle A B C, as the ſquare A F K L hath to the Triangle A F E, therefore extract the Square Root from

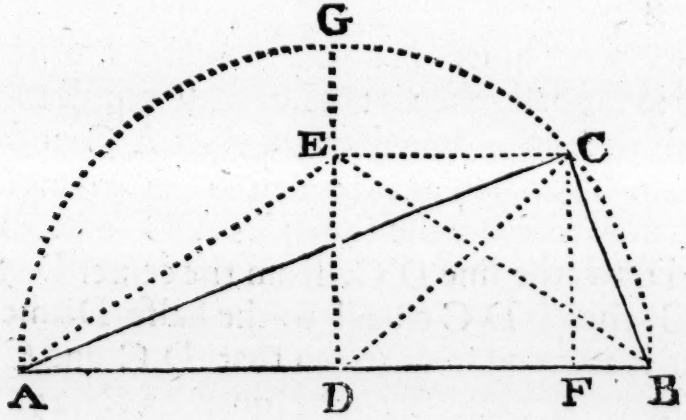
625

625, coms 25 for A F, then is F C 15: then note, that as G D is to A D, ſo alſo is C F to F A, thereby is C F known (becauſe D G is the halfe Diameter of the Circle in the Triangle A B C) That then F C is the halfe Diameter of the Circle in the Triangle A M N, therefore is F D alſo equall to F C, and ſo is F D alſo 15: Say then, If A F 25, gives D F 15, what gives A C 40? and there coms 24 for G C, then is the whole Baſe B C 48, which was required.

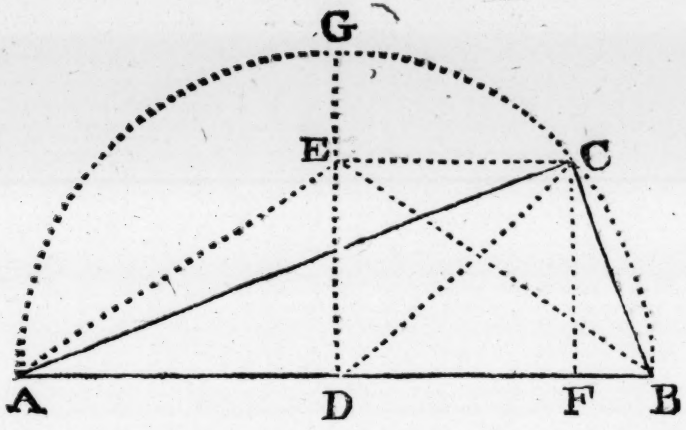
QUEST. 47.

There is a Triangle A B C, *right angled in* C, *whoſe content is 384, alſo the Hypothenuſall* A B *is 48.*

The Queſtion is; *How much are the ſides* A C *and* C B?



OF the Hypothenuſall A B make a halfe Circle, as A G C B, whoſe centre is D, and upon D erect a perpendicular D G; then double the content given, as 384, coms 768, which divide by the Hypothenuſall given, coms 16 for the perpendicular D E, of an Iſoſceles Triangle as A E B, equall to the given Triangle A B C. Then



Then draw from the point E, a line parallel to the Baſe A B, till it cut the halfe Circle in the point C, and from that point C let fall a perpendicular line upon the point F in the baſe, then is the line C F equall to D E, and is alſo 16, and if you draw the lines A C and C B, then is the Triangle A C B equall to the Triangle A E B. But now to know the two ſides A C and C B, doe thus: Draw the line D C, from the center D to the point C, then is D C equall to the halfe Diameter D G, and is 24, and ſeeing then that D C and C F are both known; if we ſubſtract the ſquare of C F 256, from the ſquare of D C 576, there ſhall reſt 320, whoſe ſquare root is $17\frac{31}{33}$, which added to 24 the halfe Diameter A D, or ſubſtracted from the halfe Diameter D B, coms for A F $41\frac{31}{33}$, and for F B $6\frac{4}{33}$, the parts of the Baſe, and F C 16, a mean proportionall between them.

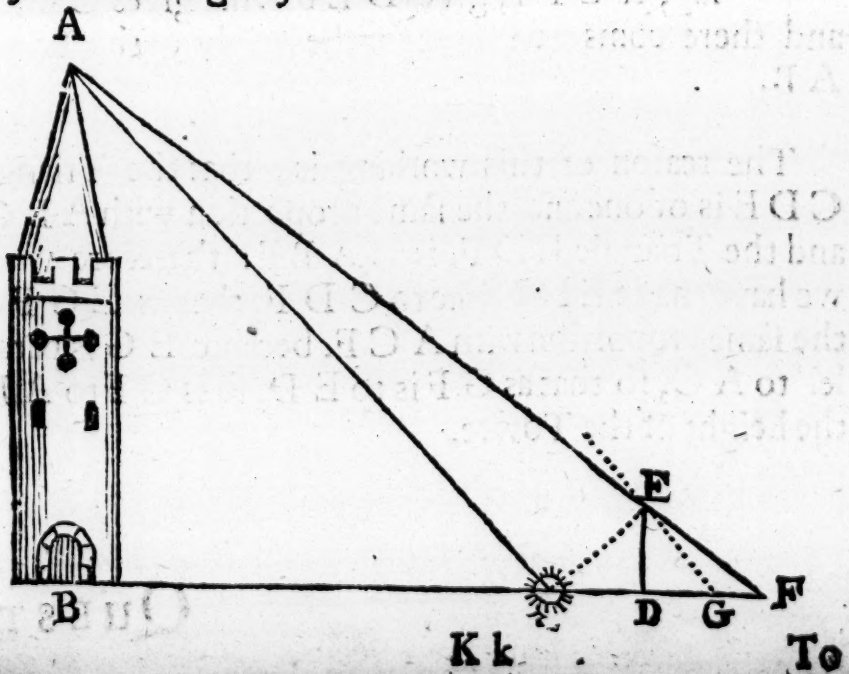
Then

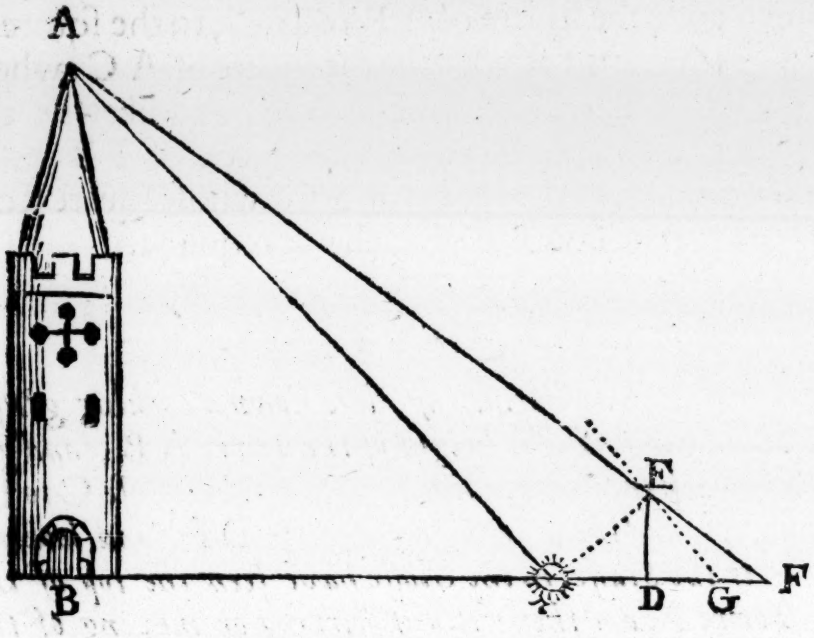
Then adde the ſquare of A F $1681\frac{961}{1225}$, to the ſquare of F C 256, coms $1937\frac{161}{1225}$, for the ſquare of A C, whoſe ſquare root is $44\frac{31}{35}$ for the ſide A C; and likewiſe the ſquare of F C 256, added to the ſquare of F B $36\frac{16}{1225}$, coms $292\frac{16}{1225}$ for the ſquare of B C, whoſe ſquare Root is $17\frac{4}{35}$ for the ſide B C, which was required.

Quest. 48.

Having (upon a plane fitting) laid down a Looking glaſſe, thereby to finde the height of the Towre A B; *and going back in the right line* B C *from the glaſſe* C *to* D, *8 foot, and there ſet up a ſtaffe as* D E, *and looking over the Staffe, in the Glaſſe have ſeen the top of the Towre* A, *and then without ſtirring or moving of the Staffe, going back yet 9 foot to* F, *ſo there I with mine eye looking over the Staffe, ſee the top of the Towre* A.

The Queſtion is; *If the ſtaffe be 6 foot long, how many foot is the height of the Towre?*





TO finde the height of this Towre, ſubſtract C D 8 from D F 9, and there reſts G F 1; then ſay, If G F 1, gives D E 6, what gives C F 17? and there coms 102 foot for the height of the Towre A B.

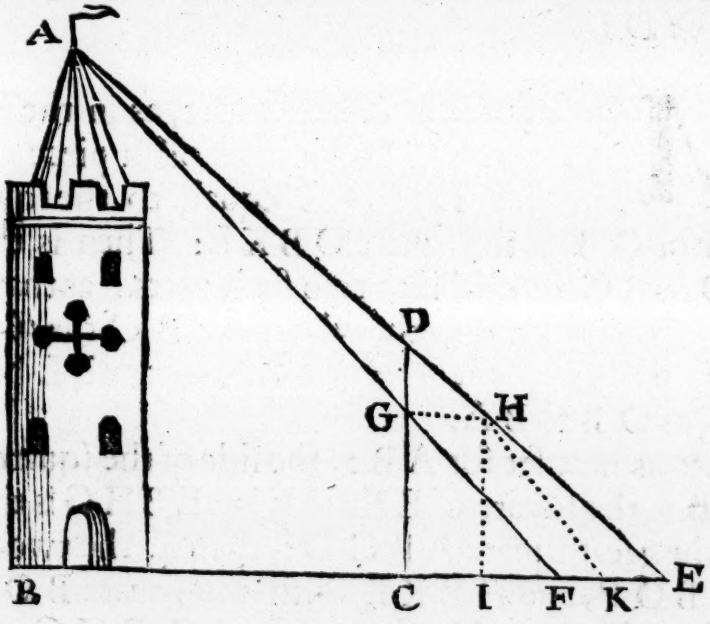
The reaſon of this working is, that the Triangle C D E is of one and the ſame proportion with A B C, and the Triangle E D F, is as A B F: therefore when we have made E D C like to C D F, then is E G F of the ſame proportion with A C F, becauſe E G is parallel to A C, ſo that as G F is to E D, ſo is C F to A B, the height of the Towre.

QUEST.

QUEST. 49.

Upon a certain place, as in the point C, *is a Staffe ſet up perpendicular in the ground, noted with* C D, *whoſe length is 20 foot, and it is found that the ſight above the Staffes top, to the top of the Towre* A B *coms to* E, *ſo that* E C *is 25 foot; and after that is found, that if the ſtaffe were foure foot ſhorter, as from* C *to* G, *that then the ſight over the ſtaffe to the top of the Towre* A *ſhould come in* F.

The Queſtion is: *If from* C *to* F (*the place of the ſtaffe*) *be 17 foot, how many foot then is that Towre high?*



SAy, If C D 20, gives C E 25, what ſhall G D 4 give? and there coms 5 for G H or F K, this taken from F E, reſts for K E 3, then ſay, if K E 3, gives H I or G C 16, what gives F E 8? and there coms $42\frac{2}{3}$, for the height of the Towre A B.

The diſtance from B to E, or from A to E may alſo be known, or from A to F, if it be required.

Quest. 50.

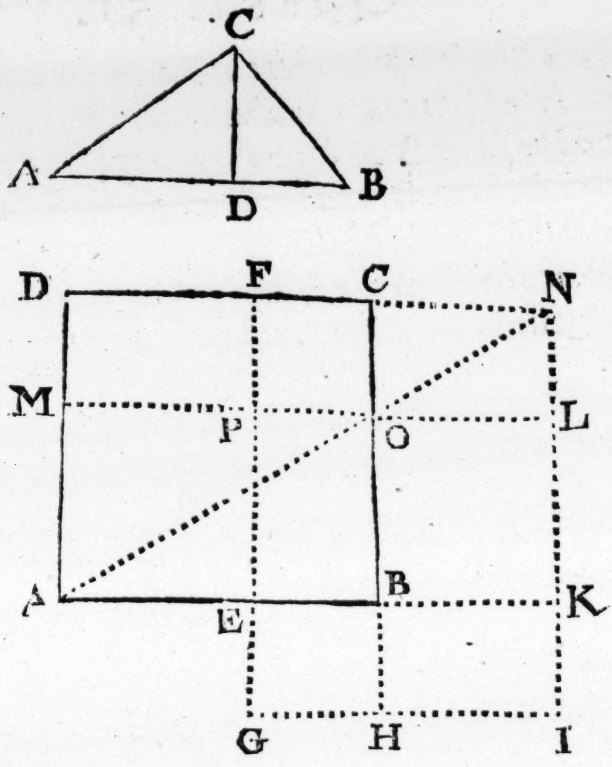
There is a Triangle, A B C, *right angled in* C, *whereof the ſide* B C *is know n* 10--- √ 20, *and that the perpendicular falling from the angle* C *upon the Baſe* B A, *in the point* D, *ſo that* B D *is to* A D, *as* A D *is to the whole line* A B.

The Queſtion is; *How much then are the ſides* A B *and* A C, *as alſo how much are the parts of the Baſe* A D *and* D B?

MAke of the line A B a Square, as in the ſecond figure A B C D; and cut off a piece of this ſquare, as D M O C equall to the ſquare of A M or O B, as the ſquare O B K L: Then is the line A D or B C divided into two ſuch parts, as that O C is to O B, as O B is to the whole line B C (*by the 11 of the 2, and 30 of the 6 of Euclid,*) becauſe L N is to O B, as O B to B A.

Let us now ſet for A B 2, the ſide of the ſquare, then is E B 1, the ſquare of E B, that is, E B H G is then 1, and the great ſquare A B C D is then 4, the two parts, as E B O P, and H B K I, with the ſquare B O L K, are then alſo 4, to this adde the ſquare E B H G, which is 1, makes together 5 for the ſquare G P L I. Then is the line G I √5, reſts then for B K or for B O (when E B 1 is taken away) √ 5—1, this √ 5—1 taken from the whole line A B or B C, reſts for O C 3—√ 5.

And



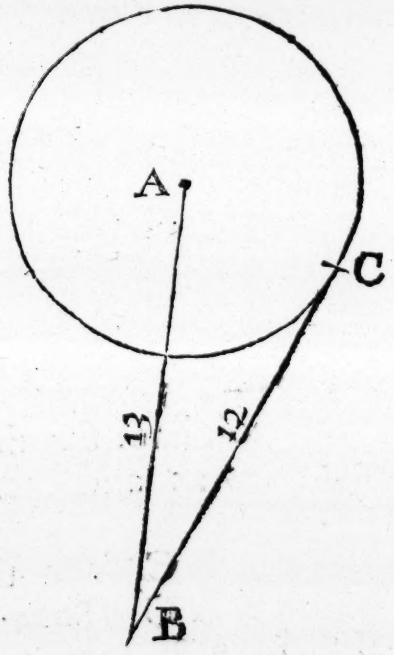
And now, As O C 3 = √ 5, is to O B √5 — 1, ſo √5 — 1, to C B 2, the whole line: Say then, If √5 — 1 gives 10 — √20, what gives 2? and there ſhall come √80 for A B the Baſe.

Take then the ſquare of B C, from the ſquare of A B, reſts for the ſquare of A C √8000 — 40: Alſo, B D is to B C, as B C to A B; then is A D as B C, 10 = √20, and B D ſhall be √180 — 10.

QUEST. 51.

There is a Circle whose centre is A, *and without the same Circle is a point taken by chance, as the point* B, *and if from the point* B *be drawn two right lines, one to the centre* A, *as* A B, *and the other from the same point* B, *which shall touch the Circle in the point* C, *as the line* C B, *so that* A B *is* 13, *and* B C 12.

The Question is; *How much is the Diameter of the Circle?*



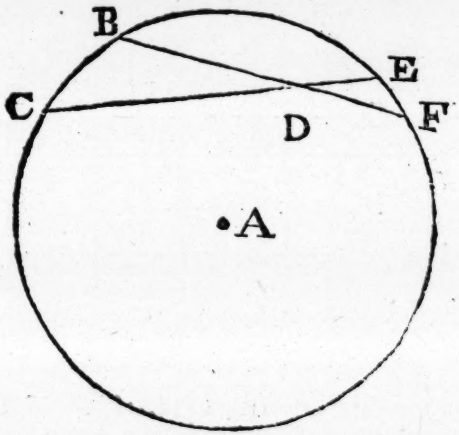
TAke the square of B C 144, from the square of A B, that is, from 169, and there rests 25, whose square Root is 5, for the halfe Diameter A C, whose double is 10 for the Diameter required.

QUEST.

QUEST. 52.

There is a Circle whoſe centre is A, *in which Circle there is two lines which doe croſſe one another by chance, and both of them doe end at the Circumference, and neither of them doe paſſe through the Centre, as theſe two lines* B F *and* C E, *and the part* B D *that is from* B *to the croſſe point* D *is* 6, *and the reſt of the ſame line, namely* D F, *is* 4, *and* D E *is* 3.

The Queſtion is; *How much is the other part* D C?



MUltiply B D 6, by D F 4, and there coms 24, then divide this product by D E 3, and there comes 8 for D C, and ſo is the whole line C E 11, and B F 10.

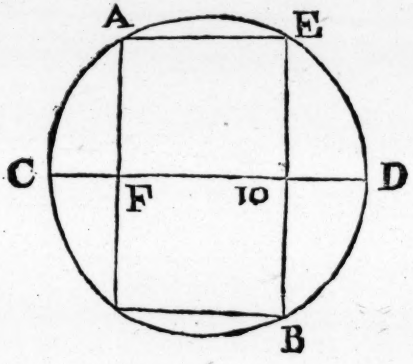
This Queſtion is proved by the 35 of the 3 of *Euclid.*

QUEST.

Quest. 53.

There is a Circle, as A E D B C, *whose Diameter is 10, and therein is required to be made a right angled figure as great as can be, whose length shall be double to the breadth.*

The Question is; *How much shall be the length and breadth of this right angled Figure?*

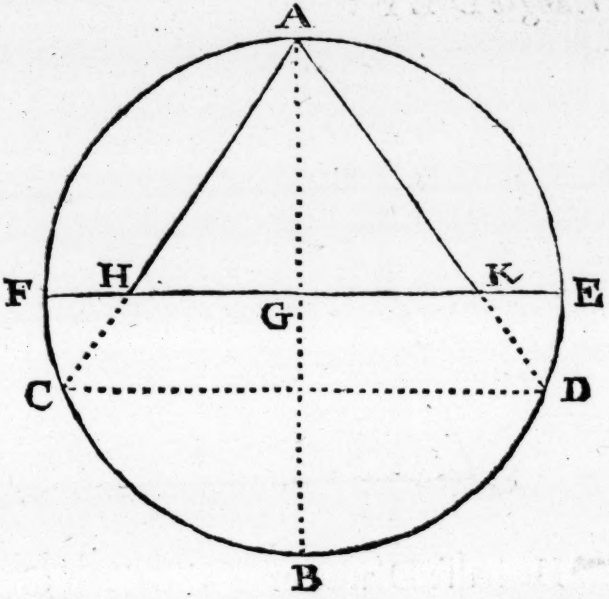


THe square of the Diameter is 100, divide this square by 5 comes 20, whose square root is the lesser side, and substract that 20 from 100, and there rests 80, whose square Root is the length of the longest side, and so there shall come √20 for A E, and √80 for E B, which was required.

Quest.

QUEST. 54.

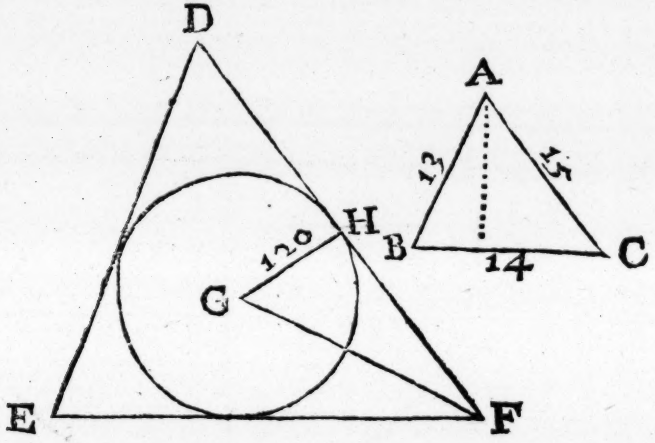
There is a Circle whoſe Diameter A B *is* 8, *It is required to make upon the Diameter of this Circle an equilateral triangle, whoſe upper angle ſhal touch the circumference*
The Queſtion is; *How much is the ſide of this Triangle?*



THe Circle given is A F B E, the one diameter is A B, draw upon the centre G another Diameter as E F, right angled upon A B, then from the point A deſcribe an Equilaterall Triangle in the whole Circle, as A C D, ſo that C D be parallel to E F, then doth the diameter E F cut off the equilaterall Triangle A H K, whoſe perpendicular A G the halfe diameter, is 4: now to know the length of one ſide, multiply 4 in himſelfe coms 16, and to this product adde one third part of the ſame product, as $5\frac{1}{3}$, and there coms $21\frac{1}{3}$, whoſe ſquare root is the ſide of the Triangle A H or A K, that is $\sqrt{21\frac{1}{3}}$ for the ſide required.

Quest. 55.

There is a Circle whose halfe Diameter is G H *120, about which is required to circumscribe a Triangle, whose sides shall be in proportion to the given Triangle* A B C. The Question is: *How much shall be each side of this Triangle* D E F?



THe halfe Diameter of the given Circle is 120, and the sides of the given Triangle is A B 13, B C 14, and A C 15, and the perpendicular (by former instructions) found to be 12, this perpendicular multiplyed by the Base B C coms 168, then adde the three sides of the Triangle given, coms 42, by this divide the said 168, the Quotient is 4, say then, If 4 gives the Base 14, what gives 120? and there shall come 420 for the Base of the Triangle required: Then say again, If 4 gives the length of the side 15 for A B, what gives 120? and there coms 450 for the longest side; and in like manner finde the third side, for which coms 390. Then if you desire a Triangle whose base is

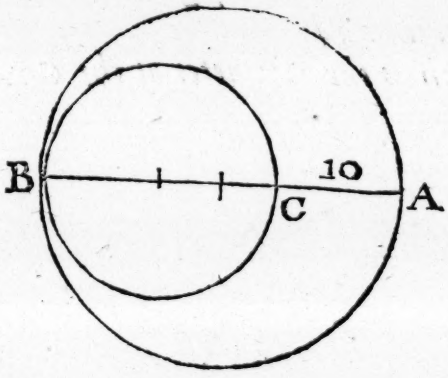
420,

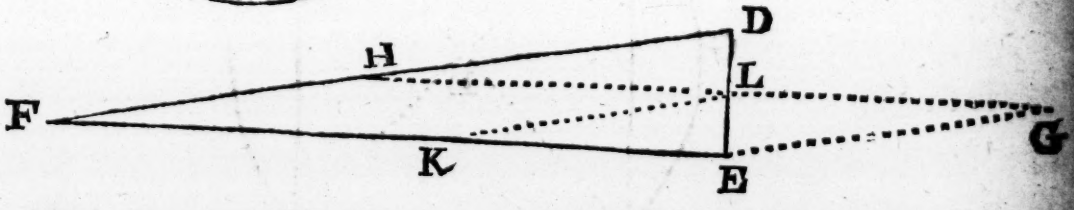
420, then are the two sides, the one 450, and the other 390, and this Triangle shall be for a Circle, whose halfe Diameter is G H 120, which was required.

QUEST. 56.

There is a Circle whose Diameter is A B, *wherein is made another Circle whose Diameter is* B C, *the difference of their Diameters is 10, and the difference of their Areas or contents is 361$\frac{3}{7}$.*

The Question is; *How much are the two Diameters,* A B *and* C B?





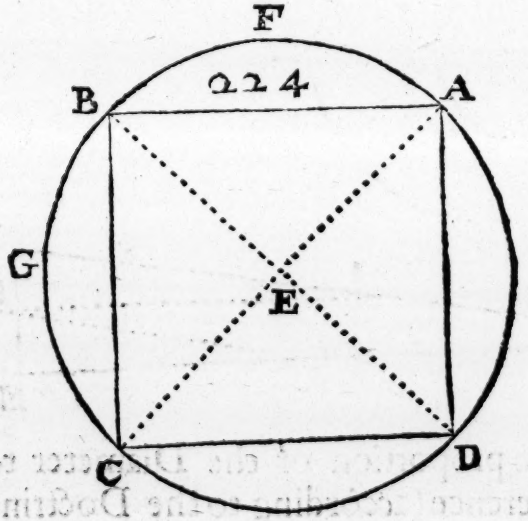
LEt the proportion of the Diameter to the Circumference (according to the Doctrine of *Archimedes*) be as 7 to 22: then in this Triangle D E F, let D E be the halfe Diameter of A B, and let the circumference be E F, the proportion then of D E to E F is as $3\frac{1}{2}$ to 22, and the difference of the halfe Diameters is E L 5, and E L H F is $361\frac{3}{7}$ the difference of their

contents, then ſay, If D E $31\frac{1}{2}$, gives E F 22, what gives E L 5? and there coms $3\ 1\frac{1}{7}$ for E K, this multiplyed by the halfe of E L, that is $2\frac{1}{2}$, coms $78\frac{4}{7}$ for the content of L E K, and ſo much alſo is E L G, this added to $361\frac{3}{7}$, is together 440, for the content of E G H F, this divided by E L coms 88 for E F equall to the circumference of the greateſt Diameter A B, then is the ſame Diameter A B 28, and the leſſer diameter C B being 10 leſſe is 18.

QUEST. 57.

In this Circle is made a ſquare of equall ſides, as ABCD, *(as great as poſſible) in ſuch ſort, that the ſegment* ABF, *or* B C G *is found to contain 224.*

The Queſtion is; *How much is the Diameter of the Circle?*



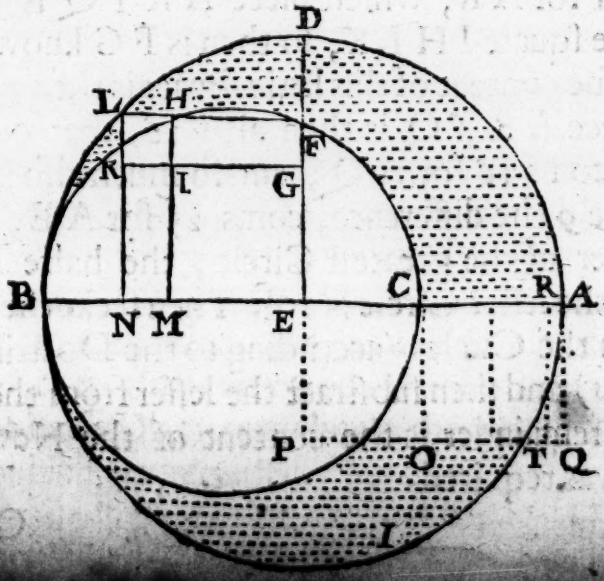
SUppoſe the diameter of the Circle be 1, and then ſeeke (by the Doctrine of *Archimedes*) the content of the Circle, and there ſhall come $\frac{11}{34}$, whereof $\frac{1}{4}$ is $\frac{11}{136}$ for the content of A E B F, ſeeke alſo the content of the

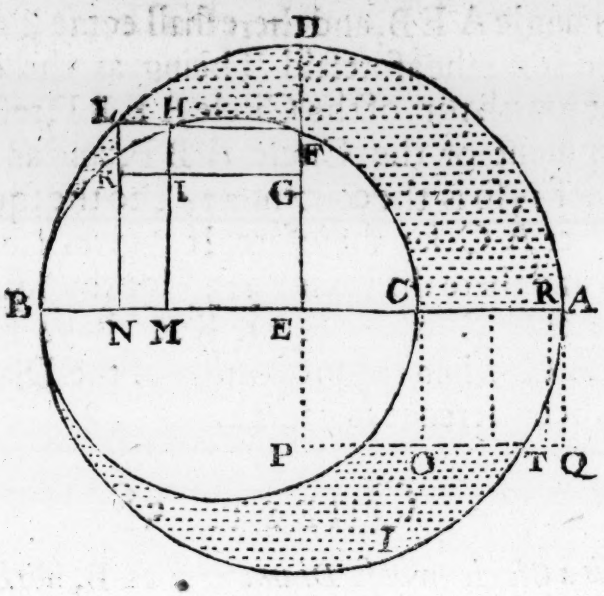
the Triangle A E B, and there ſhall come $\frac{1}{8}$, which ſubſtracted from the aforeſaid $\frac{11}{14}$ being as the content of the one fourth part of the Circle A E B F, reſts yet $\frac{1}{14}$ for the ſegment of the Circle A B F, and as $\frac{1}{14}$ is to the ſquare of 1, ſo is the content 224, to the ſquare of the Diameter A C: ſay therefore, If $\frac{1}{14}$ gives 1, what gives 224? and there ſhall come 3136 for the ſquare of the Diameter A C, whoſe ſquare Root is 56 for the Diameter A C, when the proportion of the Diameter is to the Circumference as 7 to 22.

QUEST. 58.

There is a Circle whoſe Diameter is A B, *within the ſame is made a leſſe Circle, whoſe Diameter is* C B, *touching one another in the point* B, *whereby, that part which the one Circle is greater then the other doth make the form of a New Moon, ſo that* A C *the difference of their Diameters is* 9, *and perpendicular upon the centre* E, *is drawn* E D, *and then is* F D 5.

The Queſtion is; *How much is the content of the ſame New Moon.*





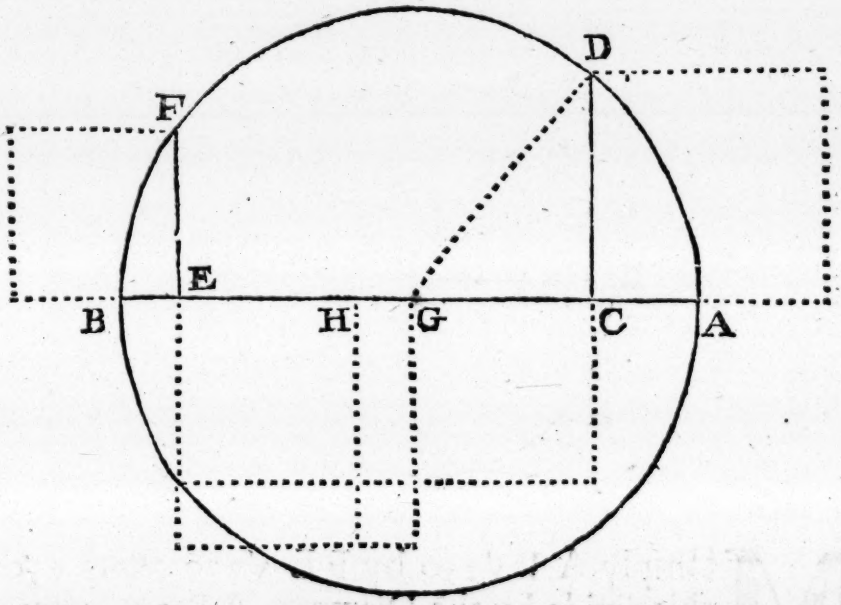
FRom A C 9, ſubſtract F D 5, there reſts 4 for F G, and becauſe F E is a mean proportional between C E and E B, therfore is the ſquare EFLN equall to the right angle figure made of A E and E C, that is, the right angle figure A E P Q: Now A Q is equall to E G or E M, wherefore adde 4 G F to 4 I K, coms together 8 for T O or R C, this taken from A C 9, reſts 1 for A R, which piece A R T Q is equall to the little ſquare I H L K, ſo then is F G known, and is 4, and the content of this little ſquare is then 16, and the little piece A R T Q is then alſo 16, that divided by A R 1, coms 16 for A Q, and ſo much alſo is E C, to this adde 9 the difference, coms 25 for A E, the halfe Diameter of the greateſt Circle, the halfe Diameter then of the leſſer Circle is $20\frac{1}{2}$. Then ſeeke the contents of both the Circles (according to the Doctrine of *Archimedes*) and then ſubſtract the leſſer from the greater, and the remainder is the content of the New Moon, which was required.

QUEST.

QUEST. 59.

In this Circle the Diameter is A B, *upon which is drawn two lines perpendicular, as* D C *being* 24, *and* E F 18, *and the diſtance between* C *and* E *is* 42.

The Queſtion is; *How much is the Diameter of the Circle* A B?



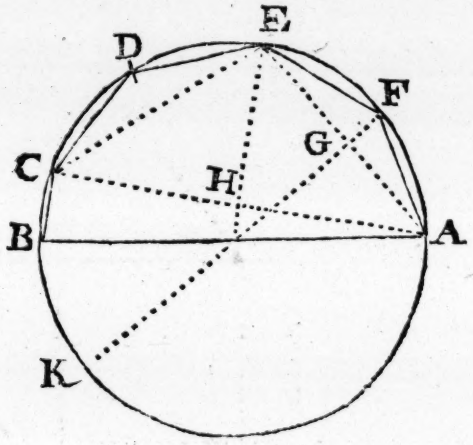
MUltiply C D 24 in himſelfe ſquare coms 576; from hence ſubſtract the ſquare of E F 18, that is 324, and there ſhall reſt 252, then multiply alſo 42 ſquare, coms 1764, from hence ſubſtract the aforeſaid 252, reſts yet 1512, which divide by the double of 42, that is by 84, and there ſhall come 18 for C G, whoſe ſquare (being 324) added to the ſquare of C D, 576, there ſhall come 900, whoſe ſquare Root is 30 for the halfe Diameter, then is the whole Diameter A B 60, which was required.

QUEST.

Quest. 60.

There is a halfe Circle whoſe Diameter AB *is* 50, *in which is drawn the line* A F *whoſe length is* √250 : *Now if this line goe four times in the halfe Circle, and be ſomthing more, ſo much as* C B,

The Queſtion is; *How much ſhall this laſt part, or laſt line* C B *be?*



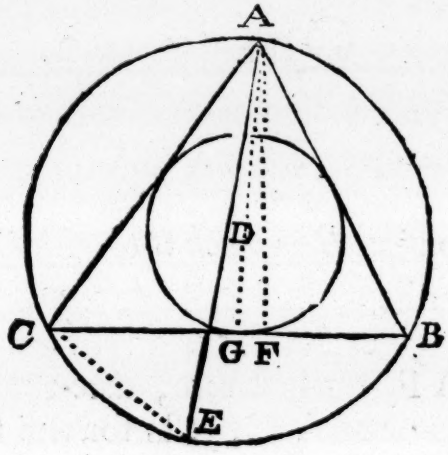
MUltiply A F √250 by F E √250 coms 250, this divide by the Diameter A B 50, coms 5 for F G, becauſe K F is to F A as F A to F G: Furthermore, ſubſtract the ſquare of F G from the ſquare of F A, reſts the ſquare of A G 225, whoſe ſquare root is 15 for A G or G E, then is A E 30, and alſo E C, in the ſame manner ſeeke E H and A H, and ſo is A C known, and there ſhall come for A C 48; this ſquare taken from the ſquare of A B 50, reſts the ſquare of B C 196, whoſe ſquare Root is 14 for the line C B, which was required.

Quest.

QUEST. 61.

There is a Circle in which is made a Triangle A B C, *whoſe ſides are* A B 13, B C 14, *and* A C 15, *and in the ſame Triangle is alſo made a Circle as great as is poſſible.*

The Queſtion is; *How much is the Diameters of both theſe Circles.*



FIrſt, to finde the Diameter A E of the circumſcribed Circle, ſeeke by our firſt Queſtion the perpendicular of the Triangle, as A F, and there ſhall come 12, and becauſe the Triangle A E C, and the Triangle B A C ſtand upon one baſe A C, therefore are the angles A E C and A B C equall *(by the 21 of the 3 of Euclid)* and for as much as the angle A C E is a right angle like as A F B, therefore is the angle B A F equall to the angle E A C, from hence it followeth, that the Triangle A C E is of one and the ſame proportion with the Triangle A F B, then is A F to A B, as A C to A E the Diameter, therefore ſay, If A F 12,

gives A B 13, what gives A C 15? and there ſhall come 16$\frac{4}{13}$ for the Diameter of the circumſcribed Circle A E.

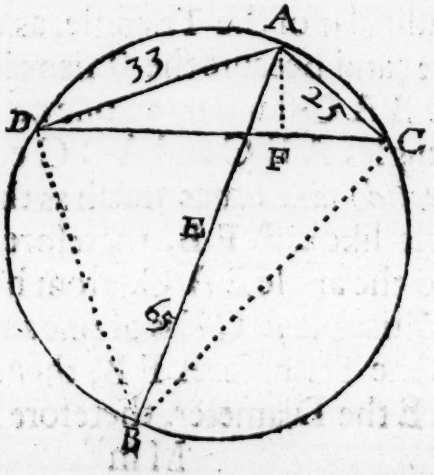
And now to finde the Diameter of the inſcribed Circle ſeeke firſt the content of the Triangle A B C, which is in this 84, the ſame doubled is 168, then adde all the three ſides coms 42, and thereby divide 168, and there coms 4 for G D the halfe Diameter of the inſcribed Circle, then is the whole Diameter 8, which was required.

QUEST. 62.

There is a Circle whoſe Diameter A B *is 65, in which is made a Triangle whereof the ſides* A C *is 25, and* A D *is 33.*

The Queſtion is: *How much is the ſide* C D?

TO perform this ſay, If A B 65, gives A D 33, what gives A C 25? and there ſhall come 12$\frac{9}{13}$, for A F, furthermore, take the ſquate of A F from the ſquare of A C, reſts for the ſquare of F C 463$\frac{153}{169}$, whoſe ſquare root is 21$\frac{7}{13}$ for C F, and ſo ſeeke F D alſo, and you ſhall finde for F D 30$\frac{6}{13}$, this added



to

to F C, coms together for C D 52: The reaſon of the working hereof for the finding of A F, is ſhewed in the former 61 Queſtion.

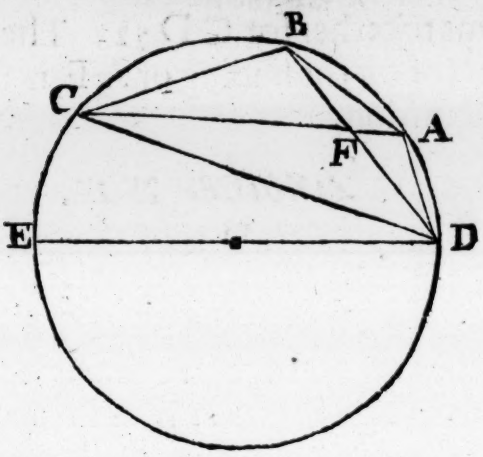
Another way.

THe Triangle A B D is of like proportion to A C F, and the Triangle A C B is of like proportion to A F D, therefore ſeeke firſt B D and B C, which may be known becauſe the angles A C B and A D B are right angles, and A B, A D, and A C, are all three of them known, and there ſhall come for B C 60, and for B D 56: ſay then, If A B 65, gives B C 60, what gives A D 33? and there coms for F D $30\frac{8}{13}$: then ſay again If A B gives 65, what gives B D 56? and there coms for C F $21\frac{7}{13}$, to this adde F D $30\frac{8}{13}$, coms together 52 for C D, as before.

QUEST. 63.

In this Circle is made a Quadrangle of unequall ſides, as A B C D, *whereof* A D *is* 16, A B 25, B C 33, *&* C D 60. The Queſtion is; *How much is the Diameter* D E, *and alſo how much are the Diagonall lines* A C *and* B D?

THe angles D A C and D B C are equall, *(by the 21 of the 3 of Euclid)* for as much as they ſtand upon one and the ſame baſe, and alſo by the ſame reaſon, the angle A D B equall to the angle A C B, and the angle A F B equall to B F C, whence it followeth that the Triangle A F D is of the ſame proportion as the Triangle B F C; ſo that A F is to F D, as F B to F C: or as F A is to F B, ſo F D to F C, wherefore we ſet for A F 1, then is B F $2\frac{1}{16}$, and F D $2\frac{2}{7}$, and F C $4\frac{19}{20}$, and B D $4\frac{37}{80}$: multiply now thoſe diagonall lines one by



another, and they ſhall produce as much as if you did multiply A D by B C, and alſo A B by D C, and then adde thoſe products together. Now A C $5\frac{19}{20}$ multiplyed by B D $4\frac{37}{85}$, doth give a product $26\frac{880}{1600}$, and AB 25 multiplyed by B C 60, coms for a product 1500; and A D 16 by B C 33 coms 528, theſe together are 2028, that ſtandeth in proportion to $26\frac{883}{1600}$ as the ſquare of A F is 10 1. Therefore divide 2028 by $26\frac{883}{1600}$, coms $76\frac{5364}{14161}$, for the ſquare of A F, whoſe ſquare Root is $8\frac{88}{169}$ for A F, then is the diagonall line A C 52, and BD 39. Now when theſe diagonall lines are found, the Triangle A B C or any of the other Triangles ſides are known, as of the Triangle ABC the ſides A B 25, B C 33, A C 52, we finde then (by our 61 Queſtion) for the Diameter E D 65. Theſe diagonall lines may very artificially be found by our 40 Queſtion, where the Triangle A B C the proportion is known, as the line B F is to (the parts of the baſe A C) A F and F C: and the two ſides A B 25, and B C 33 are known alſo, and ſo is by our 40 Queſtion known how much the baſe A C, and the parts A F and F C each apart, and A C being known we have the diameter DE known, and ſo the diagonall

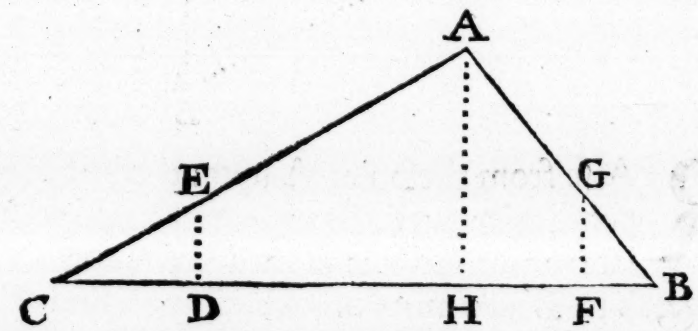
agonall D B and the parts D F and F B.

See our 41 problem for the demonſtration hereof, as alſo in *Ptolomeus* and *Copernicus*.

Quest. 64.

There is a three angled piece of land, as A B C, *whereof there is meaſured from* B *to* H, *and comming to* H *it is found that the perpendicular from the angle* A *doth fall upon the point* H, *and* B H *is 50: alſo it is found that if we meaſure from* B *to* F, *that the perpendicular* G F, *at 5 from* B, *is 12, alſo meaſure from* C *to* D *12, the perpendicular* D E *is 9.*

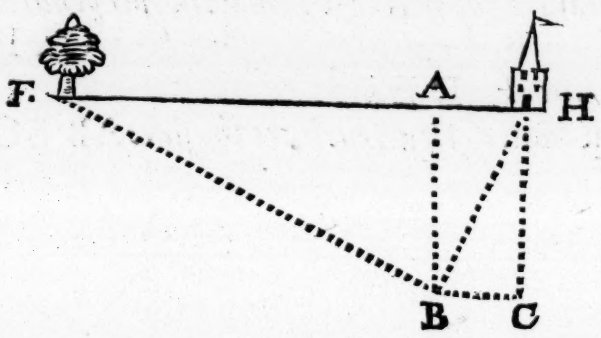
The Queſtion is; *How much are the ſides* AB, BC, *&* AC?



IF BF 5 gives FG 12, what B H 50? there coms 120 for A H, and ſo ſeeke H C, and ſay, if D E 9, gives DC 12 what gives A H 120? coms 160 for H C, which added to B H 50, coms for B C the whole baſe 210, and now that B H, A H, and H C are known, A B and AC may be known, by adding the ſquare of BH to the ſquare of AH that is 2500 to 14400, coms together 16900, whoſe ſquare root is 130 for A B: alſo add the ſquare of A H 14400 to the ſquare of H C 25600, coms 40000, whoſe ſquare root is 200 for A C.

Quest. 65.

There are two marks, (as Steeples, Trees, or such like) standing distant a certain space unknown, and being in a right line between them (the marks being F H*) and the place of standing* A*, and by reason of some certain impediments we can goe to neither of the marks, yet it is required to know the whole distance* F H*, and also the parts between* A F *and* A H.



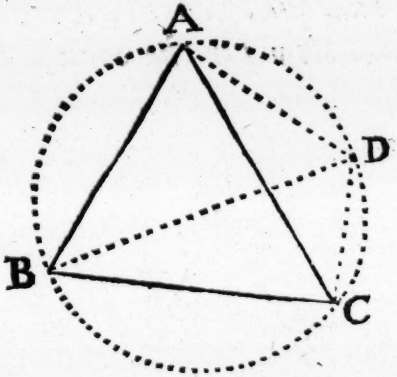
RAise from the point A upon the line F H, a perpendicular, as A B; and upon that perpendicular from the point A measure a convenient distance, as to B 500 more or lesse, till you finde that the marks F and H, with the place of your standing in B, make a right angle F B H, then see out on the one side, as to C, so that you bring A B and C into a right angle, as A B C: and then from B measure to C till you come in the right angle at C, and supposing the distance A B to be 500 and B C 200, and seeing that A H is equall to B C, therefore say, If A H 200 gives A B 500, what gives A B 500? and there shall come 1250 for A F, because the Triangle H B F is right angled in B, and A B doth fall perpendicular upon H F: then is the line H A to A B, as AB to AF. Then add AH 200, to AF 1250, and there coms 1450 for F H the distance required.

Quest.

QUEST. 66.

There is a Triangle whereof two ſides are known, as A B *52, and* B C *56, about the ſame is circumſcribed a Circle whoſe Diameter is 65.*

The Queſtion is; *How much is the third ſide of this Triangle? namely the ſide* A C.



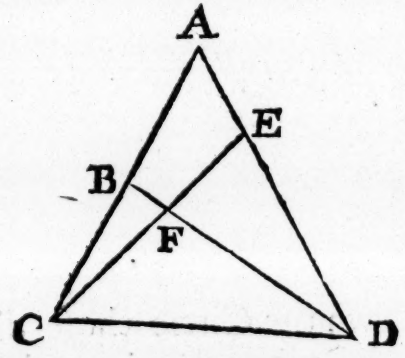
HAving drawn the Circle whoſe diameter is 65, and placed the lines therein as they were given, namely A B 52, and B C 56, then ſeeke how much the lines A D and C D are (which are eaſily found by our firſt Queſtion, in as much as two ſides of a right angled Triangle are known) and there ſhall come for A D 39, and D C 33: theſe being found, multiply A D 39, by B C 56, coms 2184: and likewiſe multiply A B 52, by D C 33, the product is 1716, theſe two products being added, coms 3900, divide the totall by the Diameter B D 65, and the Quotient will be 60 for the third ſide of the Triangle A C, which was required.

QUEST.

Quest. 67.

There is a Triangle A D C, *whose sides are known,* AC *is* 84, A D 105, *and* D C *is* √18081: *In this Triangle is drawn two lines out of the angles* C *and* D *to the opposite sides, as* E C *and* D B, *these lines crosse one another in the Triangle in the point* F, *and* B D *is* 119, *and doth divide* A C *into two unequall parts, whereof* A B *is* 56, *and* B C 28, *In like manner doth* C E 91 *divide* A D *in unequall parts,* A E 35, *and* C D 70.

The Question is; *How far the point* F *is distant from* B *or* E?



Multiply C A 84 by D A 105, coms 8820, from whence substract the product comming of B A multiplyed by A E, namely 1960, rests 6860, and this rest is your first number in the Rule of Three: Then multiply C B 28 by A D 105, coms 2940, and that is your second number in the Rule, and for the third number set the line C E, that is 91: then multiply and divide, and there shall come 39 for C F, to this adde F E 52, coms 91 for the whole line C E.

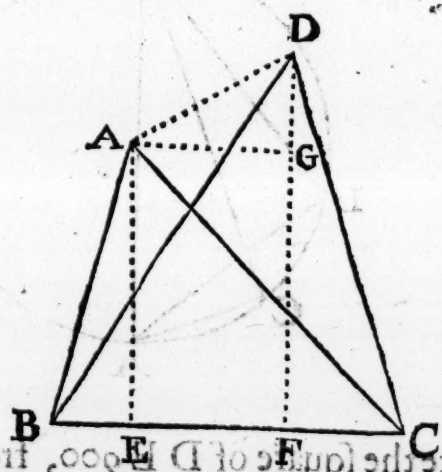
Now if you will have the parts of the line B D, then let the former divisor remain your first number, namely,

ly, 6860, and in the first place set onely the product comming of D E by A C, and in the third place set the line B D, and there shall come 102, and so much is the line F D, thereto adde F B 17, and there coms 119, for the whole line B D, and so we have found the point of cutting, which was required.

QUEST. 68.

There are two Triangles, as A B C *and* D B C, *whose sides are known for* A B *is* 30, A C 39, B C 35, B D 45, *and* D C 40, *and they both stand upon one base* B C, *and from the angle* A *to the angle* D *is drawn the line* A D.

The Question is; *How long is that line* A D?



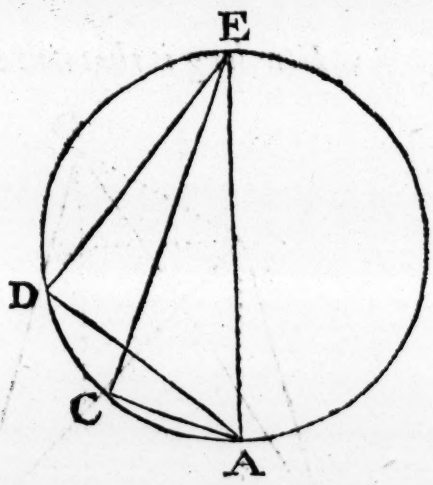
SEeke first how much is the line A C, or the perpendicular of the Triangle A B C, by our first Question, and also the parts of the base, which being known, the perpendicular is also known (*by the 47 of the 1 of Euclid.*) Then do the like with the Triangle B D C, then having found both the perpendiculars, substract A E from D F, rests D G, and so A G and D G

being known: then is the line DA alſo known *by the 47 of the 1 of Euclid.*

Quest. 69.

In this Circle A C D E, *the Diameter* A E *is unknown, upon which is made two Triangles, as* A C E, *aud* A D E, *whoſe angles* C *and* D *doe touch the Circumference, and therefore both of them right angles, and the ſide* E C *is 40, and* E D *30, and alſo the other two ſides* A C *and* A D *both together make 50.*

The Queſtion is: *How much is the Diameter?*



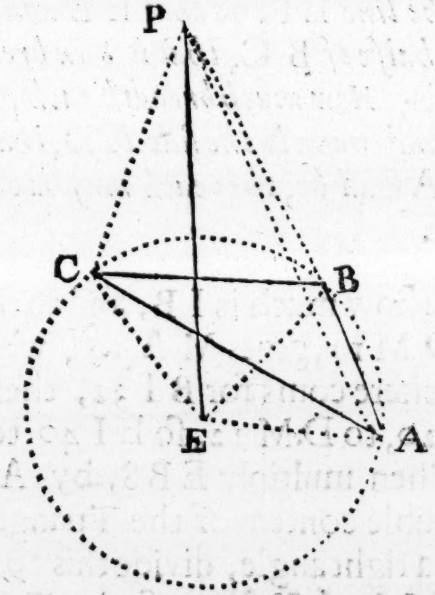
TAke the ſquare of D E 900, from the ſquare of E C 1600, reſts 700, which divided by the ſum of the addition of A C D, the Quotient is 14, which taken from 50, reſts 36, the halfe whereof is 18 for the leſſer ſide A C: Then adde the ſquare of A C 324, to the ſquare of E C 1600, coms 1924, whoſe ſquare root is $43\frac{78}{87}$ for A E the Diameter of the Circle.

Quest.

Quest. 70.

There is upon a certain place a May-pole set up, as E P, *the height whereof is 156, upon the top thereof at* P *there sits a Bird, and three young men with their Birding-peeces set themselves to shoot at the Bird, all of them equidistant from the Pole* E, *and each of them distant one from another,* B *from* C 66, B *from* A 50, *and* A *from* C 104.

The Question is; *How far each of these young men are to shoot, or, what is the distance that each of them hath to shoot from tht ground whereon he standeth, to the bird.*



FOr as much as the young men stand all of an equall distance from the Poles foot E, It is evident that the places of their standing doth make a Triangle in a Circle, therefore seeke first (*as in Quest. 61*) how much is the halfe diameter of such a Circle as A E, B E, or C E, and you shall finde it to be 65, and so much

much are their ſtandings diſtant from the foot of the Pole E, now to know how much is the diſtance of each young mans foot to the Bird (that is) how far each of them muſt ſhoot to hit the Bird, adde the ſquare of AE 4225 to the ſquare of E P 24326, coms 28561, whoſe ſquare root is 169 for the diſtance from their feet to the Bird.

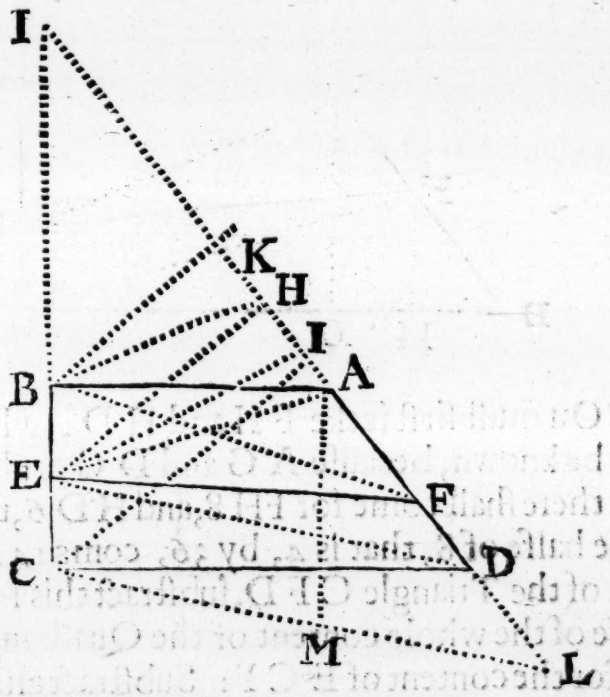
QUEST. 71.

There are two Brethren have a piece of Land, as ABCD, *whereof* A B *is parallel to* D C, *and is long* 24, D C 36, *and the angle* B *is a right angle, and* B C *is* 16, *It is required to divide this piece of Land into two equall parts with the line* E F, *ſo that* E B *and* E C *be each of them the halfe of* B C, *that is* 8 *in breadth.*

The Queſtion is; *How much breadth each part ſhall have at the other end, namely the ſide* A D, *that is, how much* D F *and* F A *ſhall be, that each may have the juſt halfe of this Land.*

FIrſt ſeeke how much is I B, and to finde the ſame ſay, If D M 12, gives M A 16, what gives B A 24? and there coms for B I 32, then is E I 40, Now as A D 20, to D M 12, ſo E I 40 to E H, coms for E H 24. Then multiply E B 8, by A B 24, coms 192 for the double content of the Triangle A B E, becauſe A B E is a right angle, divide this 192 by E H 24, the Quotient is 8 for A K, becauſe the Triangle A K E is equall to the Triangle A B E, that is that the double content of A K E is alſo 192, and alſo H E is now known, being the perpendicular upon A H, then can A K thereby alſo be known, and there coms for A K 8, and ſeeke alſo after the ſame manner D L, multiplying E C 8,

E C 8, by C D 36 coms 288, the double of the content of the Triangle E C D, becauſe C is a right angle: Now is the Triangle L D E taken equall to D C E, & is alſo his double, the content of 288, divide this by H E 24, the aforeſaid perpendicular, coms 12 for D L. Laſtly, adde D L 12, D A 20, and A K 8 together, & there coms 40 for L K, whereof the halfe is 20 for L F, or F K, from hence ſubſtract L D 12, reſts for D F 8, then is F A 12, which was required.



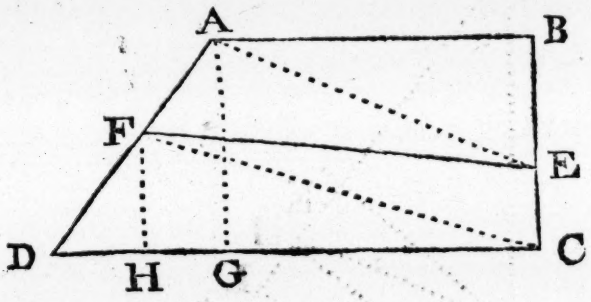
Another way.

ADde A B 24 and D C 36, coms together 60, then ſay, if 60 gives A D 20, what gives A B 24? and there ſhall come for D F 8, or what ſhall 36 give? coms 12 for F A as before.

QUEST. 72.

A Farmer hath a piece of Land, as A B C D, *of which* A B *runs parallel to* D C, *and the angles* B *and* C *are right angles, and the lines are* A B 24, B C 16, *and* C D 36: *It is required to divide this piece of Land into two equall parts, ſo that the line* E F *that divideth this land doth begin in the midſt of* A D, *ſo that* A F *and* F D *be equall.*

The Queſtion is; *How much* E C *and* E B *ſhall be?*



YOu muſt firſt ſeeke F H and H D, which may be known, becauſe A G and D G are known, & there ſhall come for FH 8, and H D 6, then multiply the halfe of 8, that is 4, by 36, coms 144 for the content of the Triangle C F D, ſubſtract this from 240, the halfe of the whole content of the Quadrangle, reſts yet 96 for the content of E C F: Subſtract alſo H D 6, from C D 36, reſts 30 for C H, the halfe is 35: further, divide 96 by 15, coms $6\frac{2}{5}$ for E C, then is E B $9\frac{3}{5}$.

This Queſtion may be anſwered after the manner of the laſt Queſtion, if you reduce the Quadrangle into a Triangle whoſe baſe is prolonged, in this B C, &c.

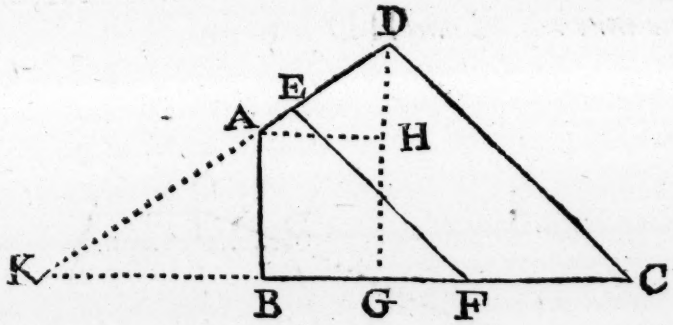
Another

Another way.

ADde A B 24, to D C 36, comes together 60, then ſay, If 60 gives B C 26, what gives 24? and there coms $6\frac{2}{5}$ for E C, or what gives 36? coms $9\frac{3}{5}$ for E B, as before.

QUEST. 73.

There is a Quadrangular piece of Land, as A B C D, *right angled in* B, *it is required to divide this piece of Land into two equall parts, in ſuch ſort, that the Line* E F *which doth divide the ſame, ſhall be parallel to* D C. The Queſtion is; *How much ſhall be* E D *and* F C, *when* A B *is* 10, B C 32, D C 25. *and* A D 13.



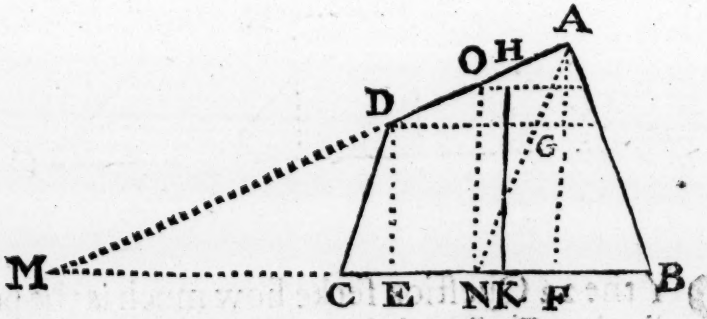
BY the 22 Queſtion ſeeke how much is the perpendiculars D G and A H, and alſo how much is D H, and there ſhall come for D G 15, for D H 5, and for A H 12. Then ſay, If H D 5, gives A H 12, what then ſhall D G 15 give? and there coms for K G 36, then is G C 20, and K C 56: Then ſeek the content of this Quadradgle A B C D, coms 300, the halfe thereof

thereof is 150, the content of the Triangle K D C 420, from hence take 150, reſts yet 270, for the Triangle K E F, then ſay, if 420 gives a Quadrat or ſquare of K C being 3136, what ſhall 270 give? coms 2016 for the ſquare of K F, then is the line K F $\sqrt{2016}$, this ſubſtracted from K C 56, reſts for F C $56-\sqrt{2016}$, and ſo ſeeke alſo E D, & there ſhall come for E D $39-977\frac{11}{14}$, which was required.

Quest. 74.

In this Quadrangle A B C D *whereof* BC *is* 50, B F 10, F E 30, E C 10, *and the perpendicular* D E 30, *and* F A 40, *It is required to divide this Quadrangle into two equall parts, in ſuch ſort, that the line* H K *which parteth the ſame, fall perpendicular upon the Baſe* B C.

The Queſtion is; *How much the ſame line* H K, *and alſo how much* K C *and* H D *ſhall be.*



IF G A 10 gives me G D 30, what ſhall E D give? and there coms 90 for E M: then multiply the halfe of E D (that is 15) by 90, coms 1350 for the content of D E M, to this adde the content of E D H K which

which is 550, is together 1900, for the content of H K M, and there ſhall come $1266\frac{2}{3}$, for the ſquare of H K, then is the line H K $\sqrt{1266\frac{2}{3}}$, and now to finde H D and K C ſay, If 1350, gives the ſquare of E M 8100, what gives 1900? and there coms for the ſquare of K M 11400, then is the line K M $\sqrt{11400}$, then is C M alſo known by reaſon that E M and E C are known, ſo that C M is 80, then is K C 11400—80. Then K H and K M being known, as alſo D E and E M, then *(by the 47 of the 1 of Euclid)* is H M and D M alſo known; coms for H M $\sqrt{12666\frac{2}{3}}$, and for D M $\sqrt{9000}$, then is H D $\sqrt{12666\frac{2}{3}}$ — $\sqrt{9000}$, which was required.

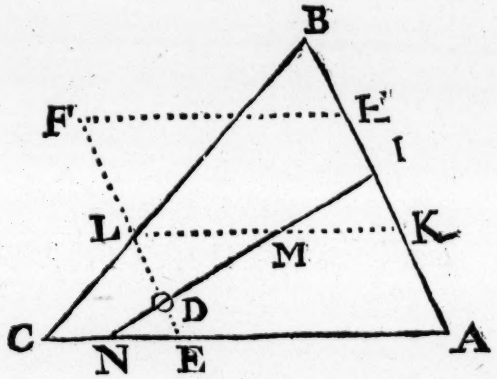
The ſame another way.

DIvide the whole content being 1400, by A F 40, coms 35 for B N, then is F N 25, ſay then, If G D 30, gives A G 10, what ſhall O P 25 (equall to F N) give? coms for A P $8\frac{1}{3}$, that taken from A F 40 reſts $31\frac{2}{3}$ for P F, and ſo much alſo is O N: Multiply O N $31\frac{2}{3}$, by A F 40, coms $1266\frac{2}{3}$, whoſe ſquare root is $\sqrt{1266\frac{2}{3}}$ for H K, becauſe H K is the mean proportionall between A F and O N.

QUEST. 75.

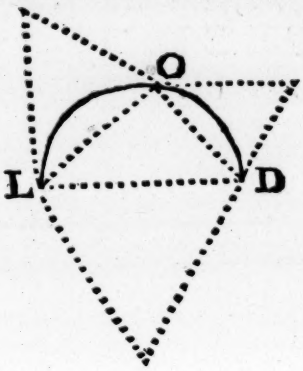
There is a Triangular piece of Land, as A B C, *of unequall ſides and angles, and in the ſame is a point or mark ſet, as* D, *through which point is required to have a right line or Ditch made in ſuch ſort, that the ſaid Triangle thereby may be divided into two equall parts.*

The Queſtion is; *How the ſame is Geometrically to be done?*



DRaw firſt a right line through the point D, parallel to A B, as the line E D F, becauſe in this it chanceth ſo that the line of diviſion muſt paſs from a point in the ſide A C, through the given point D to ſome point in the ſide A B, then make the Quadrangle H A E F equall to the Triangle A B C, and thereof take the halfe, that is then the Quadrangle K L E A, and draw a line through the given point D, ſo that the ſame line doth cut off from the Quadrangle K L E A the Triangle D L M equall to the two Triangles K M I and D E N together, Then is the Triangle I A N, yet equal to the Quadrangle K L E A, becauſe that the halfe of the given Triangle A B C (for as much as the Quadrangle K L E A is the halfe of the Triangle A B C)

A B C) is equall to the Triangle A I N, Therefore is the ſaid Triangle A B C, by the line I D N, divided into two equall parts through the point D, as was required.

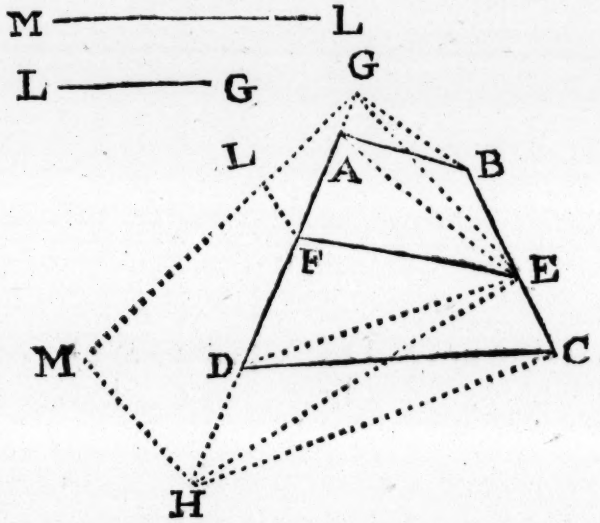


And now to draw this line N D I make a halfe Circle whoſe Diameter is the line D L, and in that end next D, ſet a line as D O equall to E D, and draw the line O L alſo: This being done, draw a line from I through the given point D, as the line I D N, which is then our line required, and doth cut through the ſide A B in the point I, and A C in N.

And for as much as thoſe Triangles I K M, M L D, and D E N, all three of them are of like proportion (which may eaſily be perceived by their angles) and the Triangle D E N, with the Triangle I K M, are together but equall to the Triangle M L D alone, therefore the ſquare of E D taken from the ſquare of D L, reſts the ſquare K I, and (*by the 31 of the 6 of Euclid*) may K I be known, then are A I and A K known, which with I N doth make the line of diviſion, and doth cut off the Triangle A I N, equall to halfe the given Triangle A B C.

Quest. 76.

There is an unequall ſided Quadrangle, as A B C D, *which is required to be divided into two parts from the point* E, *in ſuch ſort, that the one part as* A B E F, *be to the other part* F E C D, *in proportion like as the given line* G L *is to* L M.

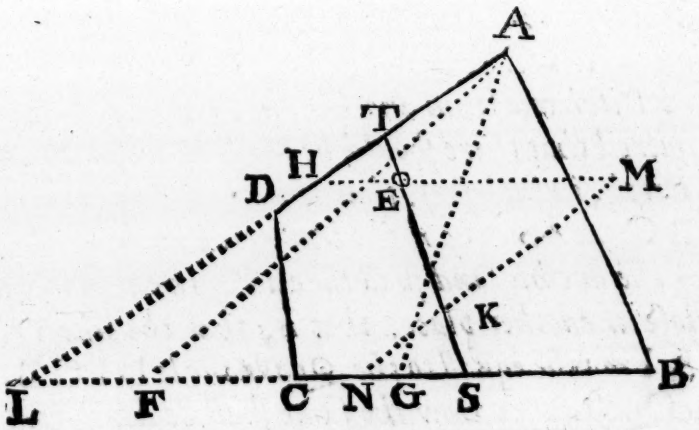


DRaw the line B G parallel to A E, and prolong D A to G, then is the Triangle A E G equall to E A B for that they ſtand upon one baſe A E, *by the 31 of the 1 of Euclid*, and ſo draw alſo C H parallel to D E, and extend A D to H, then is the Triangle E D H equall to the Triangle E D C, and the whole Triangle G H E is then equal to the whole Quadrangle A B C D. Then draw a line from the point G at your pleaſure that ſhall be as long as both the given lines G L and L M, which is in this G L M: if we draw now the lines M H, and L F parallel to M H, then is the

the line G H from the point F divided into two parts, G F and F H, which have proportion one to another, that is, G F to F H, as G L to L M, therefore the line E F being drawn, divideth the Triangle G H E into two ſuch parts, that the one part G E F is to the other part E H F as the line G L to the line L M: and as G E F is equall to A B E F, ſo is E H F equall to the part E F D C, then is the given Quadrangle A B C D divided into two parts that the part A D E F, is to the other part E F C D, like as the given line G L is to L M, which was required.

QUEST. 77.

There is an irregular Quadrangle A B C D, *therein is a point given as* E, *and it is required, that with a right line through the ſame point from the ſide* A D *to divide the ſame Quadrangle into two equall parts.*



PRolong the ſides A D and B C till they cut on another in the point L, and then make the Quadrangle into a Triangle from the angle B, that is then the Triangle A B F: this Triangle A B F divide

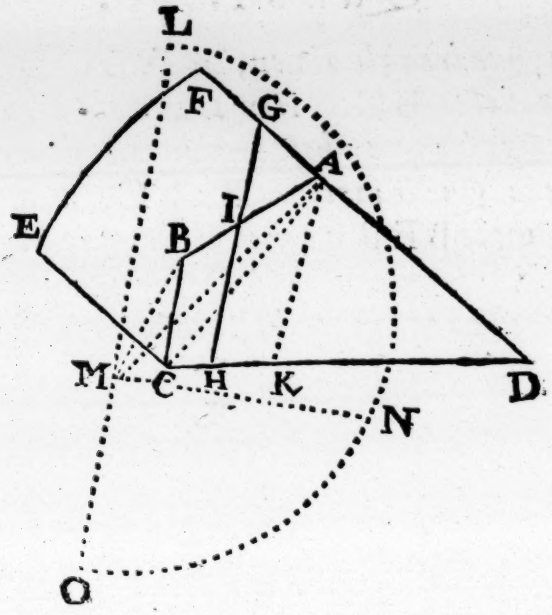
into two equall parts, as here in the point G, by the line A G, this being done, draw the line H M through the point E, parallel to the baſe B L, and make the Quadrangle H M N L equal to the Triangle A G L, in ſuch ſort, that M N be parallel to H L, which may be done diverſe wayes, then draw the line T S through the given point E, like as is ſhewed heretofore in the 75 *Queſt.* ſo that the Triangles E M K, and S K N, be together equall to the Triangle E M K alone; then is the Triangle S L T equall to the Quadrangle M H L N, or equall to the Triangle A G L, wherefore if we take away from the Triangle T S L, the Triangle D C L, then is T S C D the one halfe, and T S B A the other halfe, and T S is the line ſought for, whereby the given Quadrangle is divided into two equall parts.

QUEST. 78.

Two men have each of them a piece of Land lying next one another, the one his piece is A B C D, *the other his piece* A B C E F, *which pieces are parted with a crooked Ditch one from another, as* A B C. *Now it is required that there ſhould be made between them a right Ditch, in ſuch ſort, that this right Ditch run parallel to* B C, *as here the right line noted with* G H, *as much of the one his land ſhall take away in one place, as he ſhall loſe in another place, that is, that the like Triangle* G A I *may be equall to the Quadrangle* I H C B.

The Queſtion is; *How the ſame ſhall be done Geometrically.*

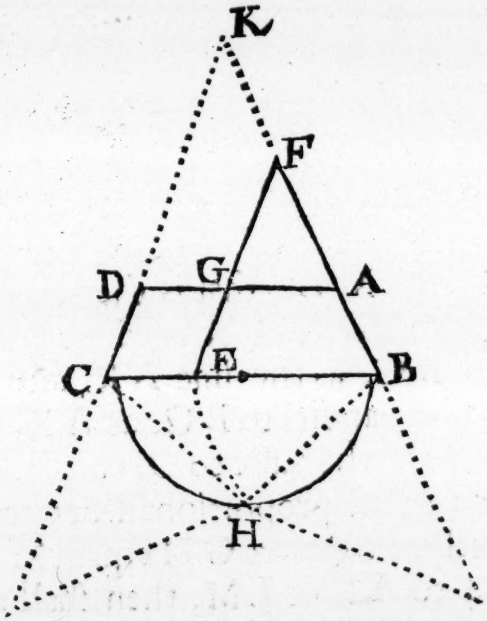
FIrſt change the quadrangle A B C D, into a Triangle, and to doe the ſame, draw B M parallel to A C, then draw a line from A to M, and ſo make the



the Triangle A M D, that is then equall to the quadrangle A B C D, this being done draw a line from A parallel to B C, as the line A K, alſo draw from the point L a line parallel to B C, as A K, which is here L M, and extend the ſame to O, equall to A K, and then is M N the mean proportionall between M O and M L, and laſtly, if we make G H equall M N, ſo that G H be parallel to A K or L M, then ſhall that quadrangle G H K A be equall to the Triangle K A M, as is ſhewed in *Queſtion 37*. Therefore is the Quadrangle G H K A alſo equall to that of A B C K, the great Triangle D G H, is then equall to the Quadrangle A B C D, or the Triangle G A I equall to I H C B, as was required. And what the one man from his piece is cut off, by the line G H, that doth he gaine againe from the other mans piece, ſo that the piece G H C E F, is yet equall to the piece A B C E F A, as before.

QUEST. 79.

There is a Quadrangle given, as A B C D, *whereof* A D *is parallel to* B C. *How to make a Triangle of this given Quadrangle whose two angles shall be equall to two of the Quadrangle, as* B *and* C, (as for example,) *this Triangle* B E F *is equall to the Quadrangle* A B C D, *and* E F *is parallel to* C D, *so that the piece* C E G D, *shall be equall to the Triangle* A C E.



MAke upon the side B C a halfe Circle, and take the length of A D, and make C H equall to A D, then draw the line B H, and make B E equall to B H, and from the point E draw a line parallel to C D, and prolong A B till they cut through one another in the point F, then is the Triangle B F E equall to the Quadrangle A D C B, as was required, or the little

little Triangle A G F equall to the Quadrangle E C D G, for if we from the Triangle B K C take away the Triangle A K D then there ſhall remain the Quadrangle A B C D.

Now is C H made equall to A D, therefore the Triangle in the ſame proportion alſo ſubſtracted from B C K, there remains upon B H a Triangle in the ſame proportion like as the Triangle B C K or A D K) *by the 31 of the 6 of Euclid)* becauſe B H C is a right angle, and alſo that B E is made equall to B H, then is the Triangle B E F (that is made of like proportion as B C K, or A D K) equall to the Quadrangle A B C D.

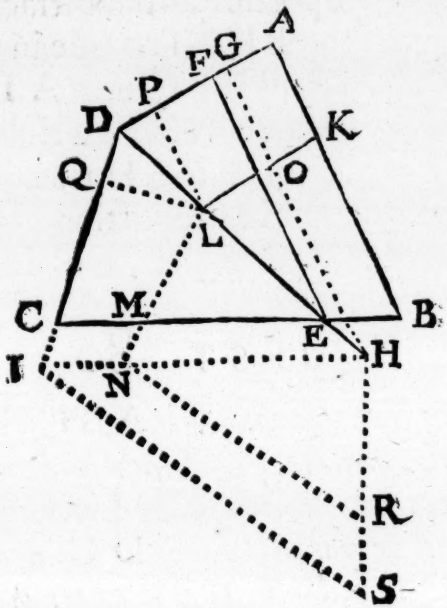
QUEST. 80.

There is an irregular Quadrangle A B C D, *which is required to be Geometrically divided into two equall parts, in ſuch wiſe, that the line* K L M *which doth divide the piece ſhall be parallel to* A D C, *and that* L P *be equall to* L Q, *and alſo that the part* B K L M, *and* A D C M L K *be each of them the halfe of the given Quadrangle* A B C D.

The Queſtion is; *How long is* A K *and* C M?

DRaw firſt the line E D, in ſuch ſort that it doth divide the angle A D C into two equall parts, then ſhall the ſame cut the line B C in the point E, then draw the line E F parallel to A B, this doth cut the line A D in the point F into two unequall parts, A F and F D, then divide the part A F in two ſuch parts, as that the one part G A, be to the other part G F, as the line D C is to F D: then prolong the line D C to I, ſo that C I be equall to A G, then is the Quadrangle

drangle G H I D of the proportion to the Quadrangle F E C D: Then make the halfe of the given Quadrangle A B C D into a right angled Square, and alſo the Quadrangle GHID into a Square, the ſide of the ſquare G H I D is H S, and if we now from the Quadrangle



G H I D, ſubſtract the ſaid ſquare of the halfe of the given Quadrangle A B C D, then ſhall remain a ſquare whoſe ſides ſhall be as here is H R: draw now S I, and R N parallel to S I, then is H I to H N, as H S is to H R, and draw N L parallel to I D, and L K to D A, then is the Quadrangle O N L H equall to the Square of H R, becauſe the Quadrangle GHID is equal to the ſquare H S, and alſo the ſquare of H S is ſo much greater then the ſquare of H R, equall to the halfe of the given Quadrangle A B C D, then is alſo the Quadrangle GHID ſo much greater then O L N H like the half of the given Quadrangle A B C D: then is alſo the Quadrangle G

HID

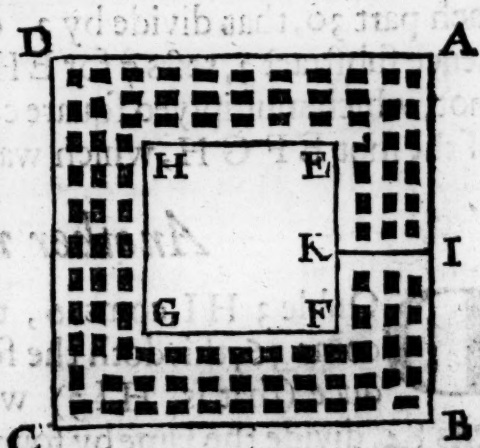
H I D ſo much greater then O L N M, like or equall to the halfe of the given Quadrangle A B C D, or O G D I N L O equall to the halfe of A B C D, and becauſe the line E D doth divide the angle A D C into two equall parts, then is L P equall to L Q alſo, in regard that C I is equall to A G, then are the parts C I N M and A G O K equall, in that they ſtand between two parallel lines, and the part K A D C M L K, like to O G D I N L O, and conſequently alſo the half of the given Quadrangle A B C D & B K L M the other half, divided by the line K L M in two equall parts, that L P is equal to L Q, which is the length of the lines A K and C M, as was required.

Quest. 81.

There is a Battalion of Souldiers ſet in a ſquare order as long as broad, in which are 256 ſhot environed with 320 Pikes.

The Queſtion is; *How many thick of men are the Pikes about the ſame to continue the perfect ſquare?*

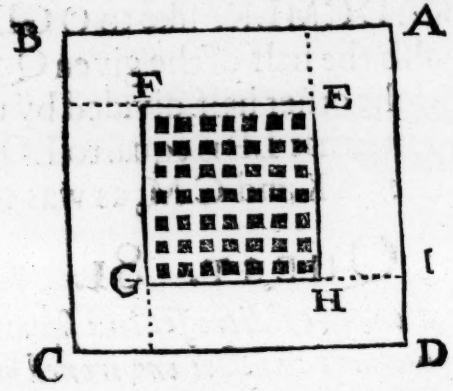
LEt there be the ſquare A B C D the full Battalion of ſhot & Pikes, 256 & 320, which is together 576 men, extract the ſquare root firſt frō 256, coms 16 for E F the ſide of the ſhot, alſo the root of 576 is 24 for A B the ſide of the whole Battalion, from hence take 16 the ſide of the ſhot, reſts 8, whoſe halfe is 4 for I K, and ſo many men thick are the ſhot environed with Pikes.



Quest. 82.

A Captain hath 120 Carbines, to which he required ſo many ſhot, as that he therewith may make a ſquare Battalion, with the Carbiners three times beſet.

The Queſtion is; *How much ſhot he muſt take to the ſame.*



LEt the whole Battalion be ABCD, & the ſquare E F G H the ſhot, which with the Light horſemen ſhall be garded three men thick, as for example H I, and now to finde the number of the ſhot, divide the number of the horſemen in four parts, coms for each part 30, that divide by 3, coms 10, for A I, from hence ſubſtract 3, reſts 7 for E H or E F, the ſide of the ſhot, which multiplyed ſquare coms 49 for the ſquare of the ſhot E F G H, which was required.

Another way.

DOuble 3 H I coms 6, that multiplyed ſquare coms 36, this doth the four corner ſquares produce (that is, H I 3) which taken from 120, reſts 84, divide the ſame by four times three, that is by

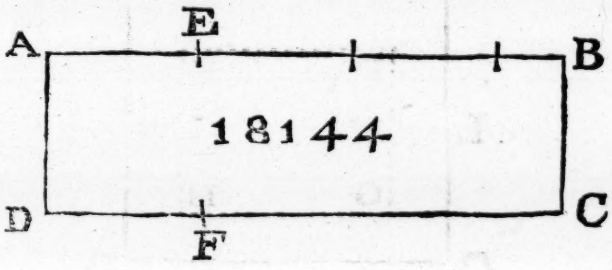
12,

12, there ſhall come 7 for E H, or the ſide of the ſquare E F G H, or the ſhot, or, they divide 84 in 4 parts coms 21, and that again by 3 divided, coms 7 for E H as before.

QUEST. 83.

There is a right angled piece of Land, whoſe length is to the breadth as 7 to 2, and doth contain 18144.

The Queſtion is: *How long and how broad is the ſaid piece of Land,* A B C D?

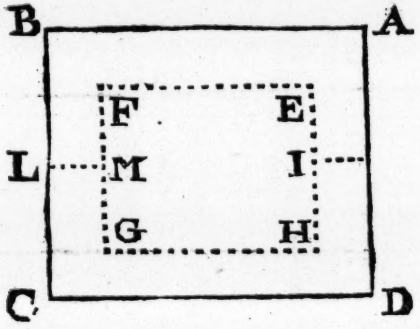


DIvide the contents in ſuch parts according as the length to the breadth, that is, if the length be 7, then is the breadth 2, therefore divide that 18144 by 7, coms 2592 for one ſeventh part, whoſe double is 5184 for A E equall to A D, whoſe ſquare root is 72 for the breadth A D or A E, which taken $3\frac{1}{2}$ times (becauſe the length is to the breadth in that proportion as $3\frac{1}{2}$ to 1, or 7 to 2) coms 252 the length, which was required.

Quest. 84.

There is a Quadrangular piece of Land, as A B C D, *whose area or content is 180, in which Quadrangle is made another Quadrangle (of the same proportion)) as here* E F G H, *whose area is 80, whereof the length of the inner part, as* E F, *is 5 lesse then the outer part* AB, *so that* I K *or* L M *are each* $2\frac{1}{2}$.

The Question is; *How long and how broad this piece of Land is?*



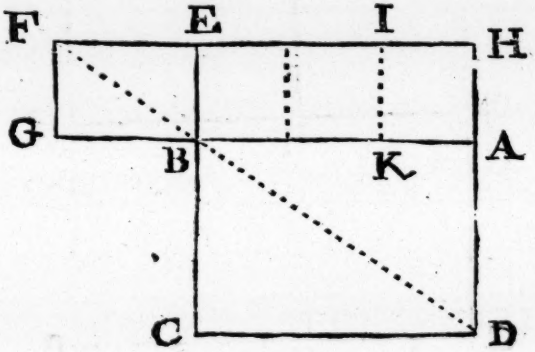
Divide 180 by 80, the Quotient is $2\frac{1}{4}$, from this extract the square Root, coms $1\frac{1}{2}$, then substract 1 from $1\frac{1}{2}$, rests $\frac{1}{2}$, then as $\frac{1}{2}$ is to 5 so is 1 to G H the length of the inner part, and also, as $\frac{1}{2}$ is to 5, so is $1\frac{1}{2}$ to C D the length of the whole land, therefore say, if $\frac{1}{2}$ gives 5, what gives 1? coms 10 for G H the length of the inner part, then is his breadth for E H 8, Also say again, $\frac{1}{2}$ gives 5, what gives $1\frac{1}{2}$? coms 15 for A B, the length of the whole ground, the breadth thereof then is 12 for B C, which was required.

Quest.

Quest. 85.

There are two right angled Quadrangles A B C D *containing 96, and* E B G F *containing 24, both of them of one proportion, the length* A B *of the one, to the breadth* B E *of the other, as 3 to 1, that is,* A B *of the one, three times* B E *of the other.*

The Queſtion is; *How long and how broad are theſe two Quadrangles?*

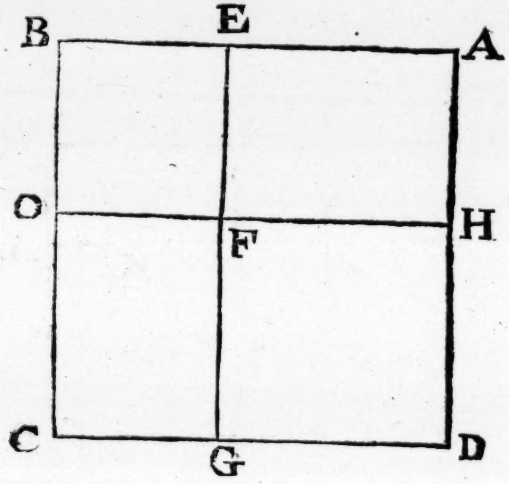


SEeke a meane proportionall between both their contents A B C D of *96* for the one, and E F G B of 24 for the other, and there ſhall come 48 for the content of the piece A B E H, of which piece A B the length, is to B E the breadth as 3 to 1, divide therefore 48 by 3 coms 16 for the ſquare H I K A, whoſe ſquare root is 4 for A H, or the breadth B E, then is the length B G 6, and the length A B three times ſo much as BE; that is, for A B 12. Then is the breadth B C 8, which was required.

Quest. 86.

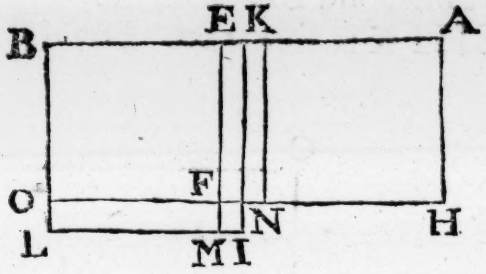
There is a Square as A B C D, *whereof the ſides are equal and each of them are 70, wherein is made two other ſquares, as* E B O F, *and* H F G D *and containeth together 2500.*

The Queſtion is; *How much thoſe two Squares* E B O F, *and* H F G D *are each apart.*



MUltiply 70 in himſelf ſquare, coms 4900, from whence ſubſtract 2500, reſts 2400, the halfe whereof is 1200 for one of the pieces AEFH, or FOCG. Then divide the line A B (as in this ſecond figure) coms 35 for B K, whoſe ſquare is 1225, for the Square K B L I, from hence ſubſtract the 1200, reſts 25 for the little Square NFMI, whoſe ſquare root is 5 for I M, that ſubſtracted from 35 B K, reſts 30 for E B, or added to 35 A K, coms 40 for A E, the ſide of the ſquare H F G D: the content then of E B O F is 900, and of H F G D 1600.

Another



Another way.

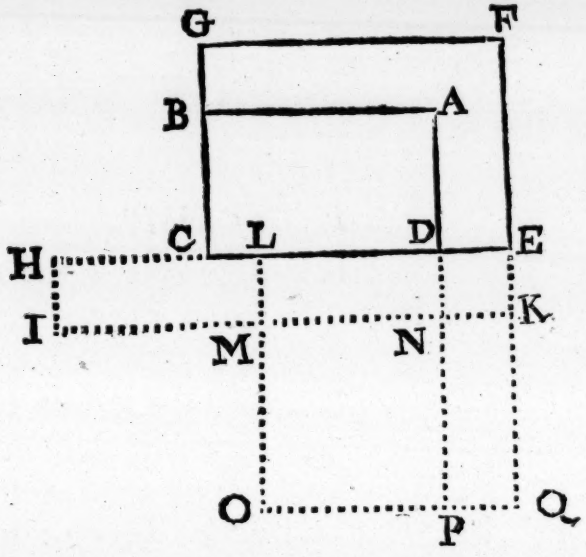
The part A E F H being 1200 is the mean proportionall between the two ſquares EBOF, and HFGD, which together makes 2500 (*which by the 5 Queſtion may be known*) now multiply the halfe of 2500 in himſelfe, coms 2562500, from hence ſubſtract the ſquare of 1200 (which is 1440000) reſts 122500, whoſe ſquare Root is 350, which being added to the halfe of 2500, coms 1600 for the ſquare HFGD, and ſubſtracted from the halfe of 2500, reſts 900 for the ſquare EBOF, as before was found.

QUEST. 87.

There is a right angled piece of Land as A B C D, *whoſe length* A B *is* 16, *and breadth* B C 10, *about which it is required to make a Fiſh-pond, beginning from* D *to* A, *and from* A *to* B, *ſo that this Fiſh-pond noted here with* D A B G F E D *contain juſt* 120, *and that* E F *be parallel to* D A, *and* F G *parallel to* A B, *and* E D *equall to* B G. The Queſtion is; *How much* D E *or* B G *ſhall be?*

MAke D C ſo much longer as A D, that C H be equall to A D 10, then is D H (equall to D A and A B together) 26, then let E K be made

equall to E D, then is the parallelogram E H I K equall to the ſaid Fiſh-pond EDABGFE, and the ſaid parallelogram E H I K is alſo 120. Divide now D I H N



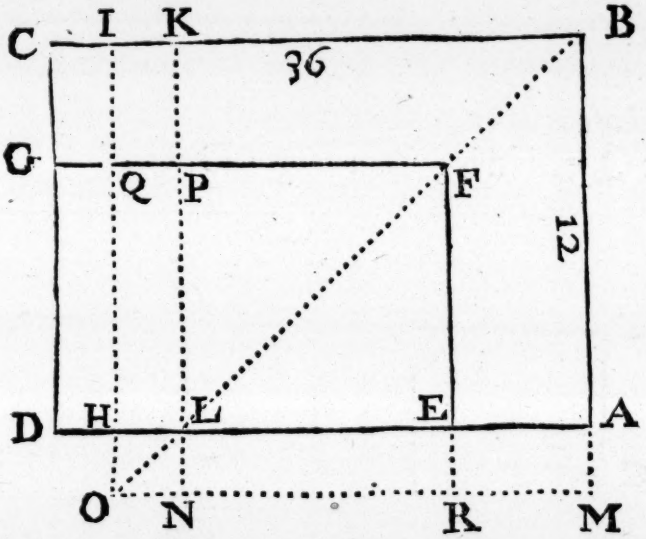
in two equall parts by the line L M, then is L D or L H each 13, by reaſon that the whole line DH is 16 and 10, that is together 26, whoſe halfe is 13 for L D or L H, then make the part K N P Q equall to L H I M, then is the Gnomon Q P N M L D E K Q alſo 120; and for as much as Q K is now equall to L M, that is alſo 13, then is the ſquare N P O M 169, this added to 120, the content of the aforeſaid Gnomon coms together 289 for the ſquare E L O Q, whoſe ſquare Root is 17 for E Q or the ſide of the ſquare, from hence ſubſtract 13 for K Q, reſts 4 for E K, and ſo much is E D or B G, which was required.

QUEST.

QUEST. 88.

There is a right angled piece of Land as A B C D, *whoſe length is 16 and breadth 12, this piece is required to be divided in two equall parts, ſo that the one ſhall be a right angled Quadrangle, and the other part a Gnomon or bound to the Quadrangle, ſo that* A E *and* C G *the breadth of the Gnomon or Square be equall and parallel to the ſides* A B *and* B C.

The Queſtion is; *How much ſhall be* A E *and* C G.



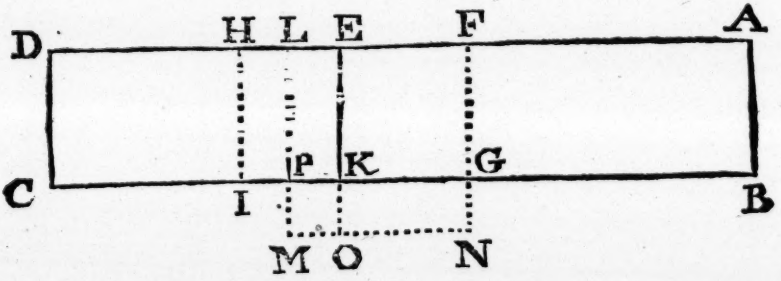
IN the ſquare ABKL, whoſe ſides are 12, then ſhall KC & LD be 4, now take half KLDC, which is IHDC, & joyn it to AL, then is ALNM equall to half IHDC, then doth want of the ſquare BIOM the little ſquare LHON, then let us take it, that the Quadrangle EFGD be half the content of ABCD, then is the content of EFGD 96, now there is taken away from EFDG the part HQGD, and that is joyn'd to EL, and ELNR in the place equal, ſo that RFQHLNR his content is yet equall EFGD 96: now adde LHON (becauſe KI is 2) then is RFQO 100, whoſe root is 10 for RO.

To this 10 adde H D 2, coms together 12 for E D, alſo take L H 2 from E H 10, reſts for E L 8, which is equall to E F or D G, and ſo is found for the length E D or F G 12, and for the breadth E F or D G 8, coms then for the content of E F D G 96, being the halfe of the whole Quadrangle A B C D, ſo that A E and G C are equall, making each of them 4, which was required.

Quest. 89.

There is a right angled piece of Land, as A B C D *which is three times as long as broad, and 12 more in length, whoſe area or content is 180.*

The Queſtion is; *How long* B C, *and how broad* A B *is?*



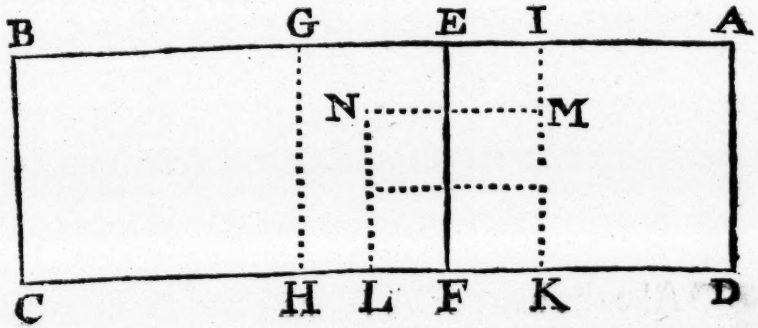
LEt A E be to A B as 3 to 1, then is E D 12, and take the $\frac{1}{3}$ of A E K B, which is F E K G, and take alſo $\frac{1}{3}$ of E D K C, that is, E H I K, then is F H I G the $\frac{1}{3}$ of the whole Quadrangle A B C D, and containeth 60, and ſeeing that E H is $\frac{1}{3}$ of E D, then is E H or I K 4, whoſe ſquate root is 2 for K P, then is the ſquare K P M O 4, which added to 60 coms 64 for the ſquare E L M N, whoſe ſquare root is 8 for F L or L M, then ſubſtract 2 for E L, reſts yet for E F 6, and ſo much alſo

alſo is AB the breadth, and A E three times 6 is 18, thereto adde 12 for E D coms 30 for A D the length required.

QUEST. 90.

There is a right angled Quadrangle, as ABCD, *whereof the length* AB *is three times as much as the breadth* A D, *and if the length were 18 leſſe then it is, then ſhould the content have been 120.*

The Queſtion is; *What was the length and breadth of this Quadrangle?*

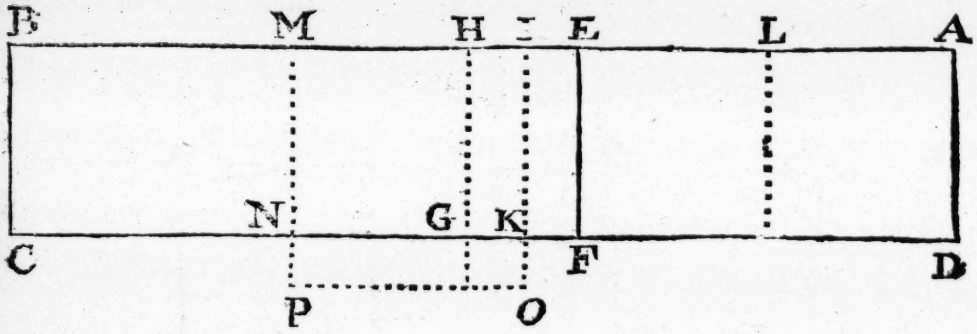


The proportion of the length to the breadth is as 3 to 1, therefore divide 18 in 3 parts, coms 6 for each part, or for E G, now let E I be $\frac{1}{3}$ of the line A E, or the ſquare I G H $\frac{1}{3}$ of the whole Quadrangle A B C D, the ſame I G H K then is 40, the halfe of G E is 3 for F L, whoſe ſquare is 9, which added to 40 coms 49 for the ſquare M N L K, the ſquare root thereof extracted coms 7 for L K, this added to F L is together 10 for KH or KI, which is equal to the breadth A D, then muſt AB be three times ſo much, that is 30, which was required.

QUEST. 91.

There is another Quadrangle right angled, as A B C D, *whereof the length* A B *is* 40, *and it is divided into two parts by the line* E F *ſo that the part* E F C B *doth contain* 192, *but* E B *is longer then* A E, *and* A E *is twice as long as* A D,

The Queſtion is; *How much then is the breadth* A D?



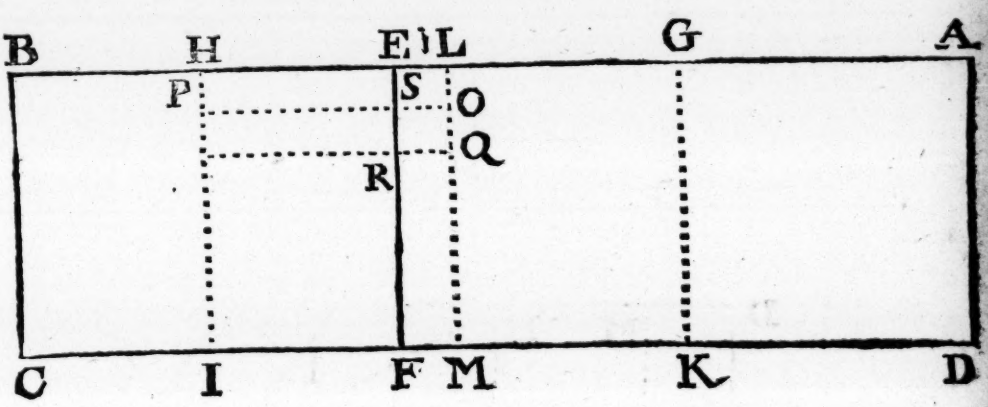
TAke the halfe of A E and E B, being together 40, coms for L M 20, becauſe A E is twice as long as A D, take alſo the halfe of 192, coms 96 for E M N F, and the halfe of 20 L M coms 10 for I M: the ſquare of 10 is 100 for the ſquare of I M P O, from hence ſubſtract 96, reſts 4 for the ſquare G K or K O, which is equall to H I, whoſe ſquare root is 2 for G K or H I, which ſubſtracted from I M 10, reſts 8 for H M, and ſo much alſo is A D, the breadth required.

QUEST.

QUEST. 92.

Like as in this aforeſaid right angled Quadrangle A B C D, *the length* A B *is known, and is* 40, *and is by the line* E F *divided into two parts, in ſuch wiſe, that the part* E B C F *his content is* 192, *and* E B *is ſhorter then* A E, *and* A E *is twice as much, or as broad, as the breadth* A D.

The Queſtion is; *How much then is the breadth* A D?



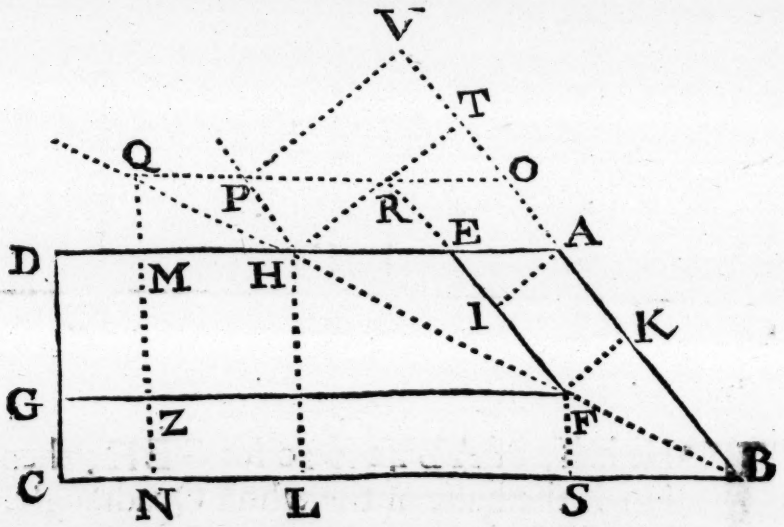
THe halfe of A B 40 is 20 for G E H, then is G H I K the halfe of the whole Quadrangle, and E H I F is then 96; in regard that E H I F is the halfe of E B C F, then take the halfe of G H, that is L H or M I, that is 10, and ſquare the ſame, coms 100 for the ſquare of M I P O, from hence ſubſtract 96, reſts 4 for the ſquare O Q R S, whoſe ſquare Root is 2 for O Q or L E, which added to M O 10 equal to M I or L H, and makes together 12 for L M, and ſo much alſo is A D the breadth required.

QUEST.

Quest. 93.

There is a Quadrangular piece of land A B C D, *whereof the ſides* A D *and* B C *are parallel one to another, and the angles* C *and* D *are right angles, the ſides are* A D 38, D C 16, *and* B C 50: *It is required to divide this piece of land in two equal parts, in ſuch manner that the lines of diviſion be parallel to the ſides* A B *and* B C, *and that* A I *and* K F *be equall to* F S *or* C G.

The Queſtion is; *How much* A E *and* C G *are?*



MAke A H equall to A B, now ſeeing that C D, A D, and B C are all known, then may A B be known, and is 20, then draw the line B H Q in ſuch ſort that the angle B be divided into two equall parts, then is A H alſo equall to A B 20, and H D 18, this H D muſt now be ſo divided in two parts, as that H M be to M D, like as B L to B A, and ſeeing that L C is made equall to H D being 18, then is L B 32,

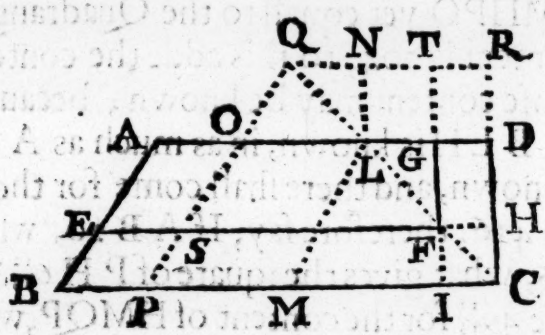
There-

Therefore ſay, 20 and 32 (makes together 52) gives 18, what gives 20? and what ſhall 32 give? and there ſhall come of 20 for M D $6\frac{12}{13}$, and of 32 coms for M H $11\frac{1}{13}$, farther more, let us make the depreſſed Parallelogram A O P H, which is equall to N M C D, for as much as T H is equall to H L, and H P equall to M D, as by the former diviſion, and by that the Triangles H T B and H L B be equall angles, and in as much as T H P V ſtandeth between two parallel lines A V and H P, therefore is A O P H equall to T H P V or N M D C; then is O B N M H P O yet equall to the Quadrangle A B C D whoſe content is 704, to this adde the content of H M Q P, whoſe content may be known, becauſe the content of A B L H is known, in as much as A H, H L, & B L are known, and there ſhall come for the content of A B L H 416, therefore ſay, If A B 20, whoſe ſquare gives 416, what gives the ſquare of P H $6\frac{12}{13}$? and there ſhall come $49\frac{11}{13}$ for the content of H M Q P, which added to 704 coms together $753\frac{11}{13}$ for the content of O B N Q, from hence take 352 the halfe, reſts $401\frac{11}{13}$ for R F Z Q: ſay alſo, If A B 20 gives C D 16, what gives P H $6\frac{12}{13}$? coms for M Q $5\frac{7}{13}$, then is Q N $21\frac{7}{13}$, this ſquare multiplyed comes $463\frac{153}{169}$, ſay then again, if $753\frac{11}{13}$ gives $463\frac{153}{169}$, what gives $401\frac{11}{13}$? coms $247\frac{49}{149}$ for the ſquare of Q Z, then is the line Q Z $\sqrt{247\frac{49}{149}}$, which ſubſtracted from Q N $21\frac{7}{13}$, reſts $21\frac{7}{13} - \sqrt{247\frac{49}{149}}$ for Z N, and ſo much alſo is C G, and laſtly ſay, if 16 gives 20 what gives $21 - \sqrt{247\frac{49}{149}}$? coms $26\frac{22}{31} - \sqrt{386\frac{66}{169}}$ for A E which was required.

Quest. 94.

There is a Quadrangle, as A B C D, *whereof* A D *is parallel to* B C, *and the angles* C *and* D *are right angles; the sides are* A D 38, D C 16, B C 50, *this Quadrangle is required to be divided in two equal parts that the line of division shall goe parallel to the lines* B C *and* C D, *so that* F I *be equall to* F H.

The Question is; *How much is* G D, *and* B E?



TAke D C 16 from D A 38, rests for L A or M B 22, then is M C 28, which added to D C 16 makes together 44, then say, If 44 gives 22, what gives 16? also what gives 28? and there shall come of 16 for O A 8, and of 28 coms 14 for O L, then is this line A L divided into two parts, so that O A is to O L, like as M C to C D, therefore make C D so much longer, as A O, then is C R 24: adde also M P 14 to M C, coms together 42 for P C; this done, wee seeke the content of the whole Quadrangle A B C D, and thereof we take the halfe, which is 352, also wee must seeke the content of C R Q P, and we finde 792, from hence substract 352, being the halfe of the given Quadrangle ABCD, rests yet 440 for the content of S Q T E: then multiply C R 24 square, coms 576, then say,

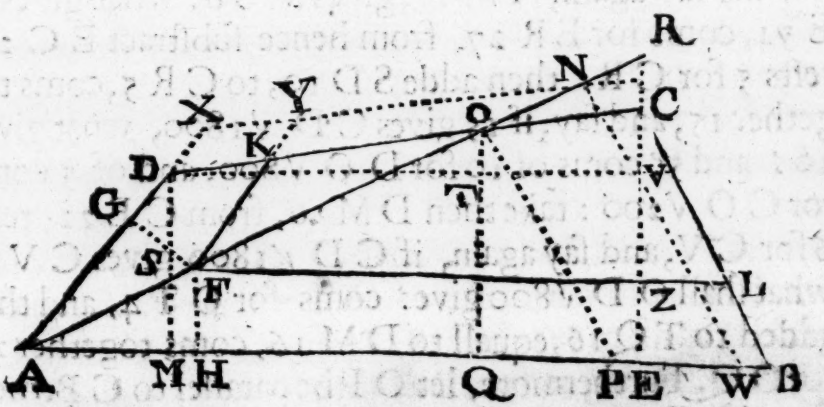
ſay, if 792 gives 576, what gives 440? there ſhall come 320 for the ſquare of F T, then is the line F T √320, this taken from T I being 24, reſts for F I 24--- √320, and ſo much is F H or G D, which we were to finde.

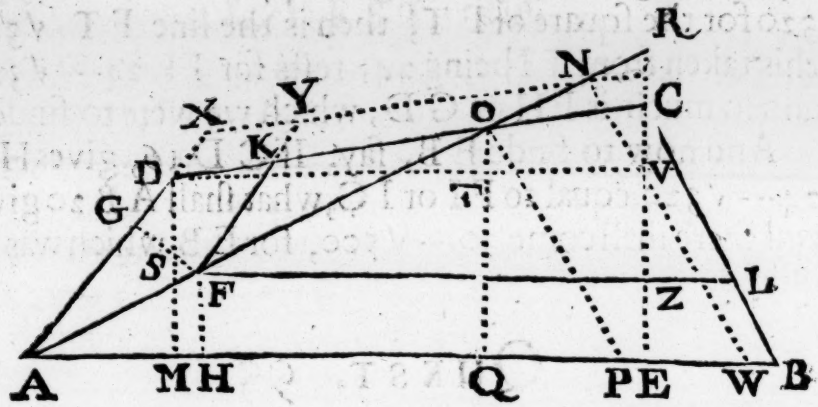
And now to finde E B, ſay, If C D 16, gives H C 24--- √320, equal to F I or I C, what ſhall A B 20 give? and there ſhall come 30--- √500, for E B which was required.

Quest. 95.

There is a piece of Land as A B C D, *whoſe ſides are neither equall nor parallel one to another, neither are their angles one equall to another, only their ſides are known,* A B *is* 65, B C √605, C D √1800, *and* D A 20, *and the perpendicular* E C 12; *It is required to divide this piece in two equall parts, in ſuch manner, that* K F L *be parallel to* D A B, *and that* F G *and* F H, *be equall one to another.*

The Queſtion is; *How much ſhall* F G *or* F H, *as alſo* D K *and* L B *be?*





SEeke firſt how much is B E, which (becauſe C E is known) is found *by the 47 of the 1 of Euclid*, and there ſhall come for B E 11, then is E A 54, ſeeke alſo the perpendicular D M, *as in Queſtion 21*, and you ſhall finde for D M 16, the content then of the propounded Quadrangle A B C D is known, then adde A M and A D together, coms 32, and ſay, if 32 gives D M 16, what gives A D 20? and A 12? and of 20 coms for D S 10, and of 12 for S M 6 *(by the 3 of the 6 of Euclid)* ſay again, A M 12, gives M S 6, what gives A E 54, coms for E R 27, from hence ſubſtract E C 22, reſts 5 for C R: then adde S D 10, to C R 5, coms together 15, and ſay, if 15 gives C D √1800, what gives 10? and 5? coms of 10 for D O √800, and of 5 coms for C O √200: take then D M 16, from C E 22, reſts 6 for C V, and ſay again, if C D √1800 gives C V 6, what ſhall O D √800 give? coms for O T 4, and that added to T Q 16, equall to D M 16, coms together 20 for O Q. Furthermore, let O P be parallel to C B, then ſay, if E C 22 gives E B 11, what gives Q O 20? coms for P Q 10, and the ſquare of P Q added to the ſquare of Q O, coms for the ſquare of O P 500, then is the line

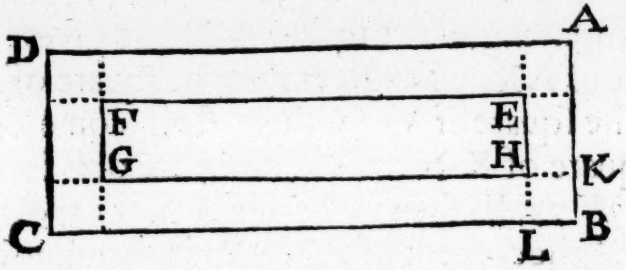
O P

O P √500, alſo is T D known, and is equal to Q M 28, then is A P 50, then is there now theſe four lines, A P 50, P O √500, O D √800, and A D 20, and fourthly, we finde B L and D K as followeth, ſeeke firſt the content of A D O P which is 700, and alſo the content of the whole Quadrangle A B C D is 1015, whoſe halfe is $507\frac{1}{2}$, and now divide B P 15 in two parts, having proportion one to another as A D to A P, coms then for W B $4\frac{2}{7}$, and for W P $10\frac{3}{7}$; then we adde W B $4\frac{2}{7}$, to A D, coms $24\frac{2}{7}$ for A X, and A V is $6\frac{5}{7}$, and A X N W of like proportion as A D O P, then can N X and N W be known, ſaying, if the ſquare A D 400 gives $589\frac{29}{49}$, the ſquare of A X, what gives 800 the ſquare of O D? and 500 the ſquare of O P? there ſhall come of 800 for the ſquare of K N $1179\frac{29}{49}$, and 500 coms for the ſquare of N W $737\frac{12}{49}$, then is the line X N √$1179\frac{29}{49}$, and N V √$737\frac{12}{49}$, the content of A N X W is then known alſo, by ſaying, if 400 the ſquare of A D, gives 700 the content of A D O P, what ſhal $589\frac{19}{49}$ the ſquare of A X give? coms $1032\frac{1}{7}$ for A X N W from hence take half the content of A B C D, reſts $524\frac{2}{14}$ for the content of F Y N Z, then is N Y and N Z known, becauſe the proportions of them are known: ſay again, if 700 gives the ſquare of O D 800, and the ſquare of O P 500, what ſhall the content $524\frac{2}{14}$? coms for the ſquare of N Y $599\frac{24}{49}$, & for N Z $374\frac{79}{98}$ whoſe root is for N Y √$599\frac{29}{49}$, and for N Z √$374\frac{79}{98}$, this N Y taken from N X reſts Y X √$1179\frac{29}{49}$ — √$599\frac{12}{49}$, and ſo much alſo is K D; ſubſtract alſo N Z from N W, reſts √$737\frac{12}{49}$ — √$374\frac{79}{98}$ for Z W, and ſo much alſo is L B, thus we have found K D and L B; and now to finde F H ſay, if C B √605, gives 22 for E C, what gives L B √$737\frac{12}{49}$ — √$374\frac{79}{98}$ coms $24\frac{3}{7}$ — √$299\frac{29}{49}$ for F H or F G which was required.

Quest. 96.

There is a four ſquare Battalion ſet in order, three times ſo long as broad, whereof the Pikes are invironed with ſhot four men thick, round about, as here you ſee in this figure the breadth A B, *and the length* B C, *the Pikes are placed within the inner ſpace* E F G A, *the outermoſt room is the ſhot, in number 576.*

The Queſtion is; *How many Pikes muſt there be to make the Battalion compleat?*

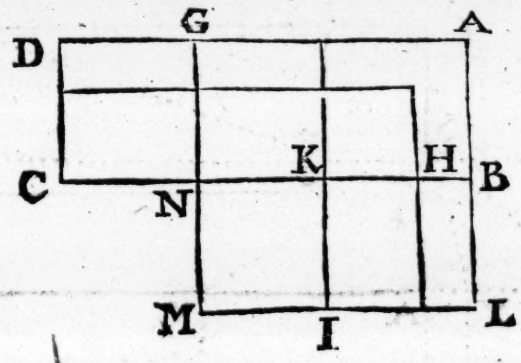


ADde to the number of ſhot four times ſo many as the ſquare of 4, that is, four times 16, (becauſe the ſhot are four men thick, and each of thoſe at the foure angles, as K H L B) and there coms 64, which added to the ſhot coms 640, which divided by 4 (the thickneſſe of the ſhot, as B K or H K, coms 160, whoſe halfe is 80 for the breadth of the front B A and the flank B C together, and for as much as A B is to B C as 4 to 3, therefore adde 1 and 3 coms 4, and ſay, if 4 gives 80, what gives 1? coms 20 for A B, and what gives 3? coms 60 for B C the length: and now the length and breadth of the Battalion being known, then is the content or number of the whole Troop 1200 men, from hence ſub-

ſubſtract 576 the number of ſhot, reſts 624 men, the number of Pikes, which was required.

QUEST. 97.

If the number of the ſhot (above ſaid) be known to be 576, and that they muſt inviron the Pikes four men thick, about the aforeſaid number of Pikes (being 624) and the proportion of the length A D, *to the breadth* A B, *were unknown, how as then to finde* A B *and* A D.



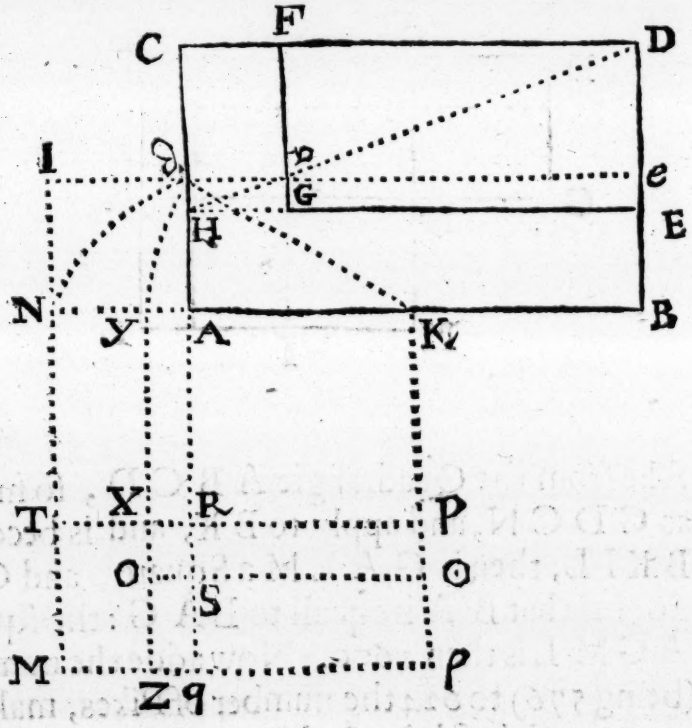
TAke from the Quadrangle A B C D, ſo much as G D C N, and apply to B K, and is become B K I L, then is G A L M a Square, and G A or A L 40, for that B A is equall to L A G: the ſquare then of A G M L is then 1600. Now adde the number of ſhot (being 576) to 624 the number of Pikes, maketh together 1200, that ſubſtracted from 1600, reſts 400 for the ſquare K N M I, whoſe ſquare root is 20 for I K, which is equall to B L, then is B L 20, that taken from L A 40 reſts for A B 20, then muſt A D be 60, becauſe that G D is equall to B L.

QUEST.

QUEST. 98.

To divide a right angled Quadrangle in two or more equal parts proposed with two lines parallel to two sides, after another manner demonstrated.

The right angle Quadrangle given is A B C D, *it is required to divide the same in two equall parts, with lines parallel to the two sides, that is,* F G *parallel to* A C, *and* G E *parallel to* A B, *that the breadth* C F *be equall to* B E.



DIvide A B in two equal parts in the point K, and also the side A C in two equal parts in the point D, and prolong the line A B to N, and also prolong the line *e d* to I, so that drawing the line I N, the same make a square, as A *d* I N: then from the point K, with

with the diſtance *d* K, draw the arch *d* N then I ſay that Y N is the breadth required, with which diſtance draw the lines G E and G F, and ſo make the Gnomon A B E G F C which contains the halfe of A B C D.

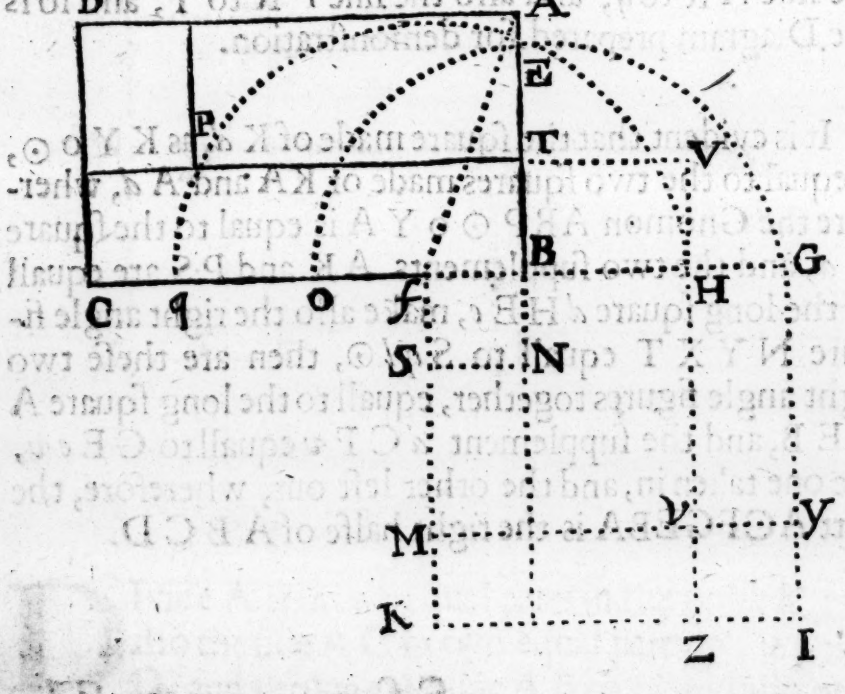
Demonſtration.

FIrſt draw the occult line *d e*, and produce it to I, and make the ſquare I *d* A N; make alſo upon A K the ſquare A R P K, make likewiſe the ſquare K N M *l*, and draw the line Y *z*, and the ſquare K Y o ⊙, and extend the line A R to *q*, and alſo the line P R to T, and ſo is the Diagram prepared for demonſtration.

It is evident that the ſquare made of K *d*, as K Y o ⊙, is equal to the two ſquares made of K A and A *d*, wherfore the Gnomon A R P ⊙ o Y A is equal to the ſquare A *d*, and the two ſupplements A K and P S are equall to the long ſquare *d* H E *e*, make alſo the right angle figure N Y X T equall to S *q l* ⊙, then are theſe two right angle figures together, equall to the long ſquare A H E B, and the ſupplement *d* C F *v* equall to G E *e v*, the one taken in, and the other left out, wherefore, the part A C F G E B A is the right halfe of A B C D.

The ſame another way.

LEt the right angled Figure be A B C D, and let the ſide B C be prolonged to G, ſo that B G be equall to B A, then divide the ſide B C in two equall parts in o, and then ſeeke a mean proportionall line between o B and B G, which let be B E, then divide the whole line C G in two equall parts, the midſt is *f*, and with the diſtance *f* E draw a halfe Circle from the centre *f*, as H E *q*, cutting off the part C *q*, for the breadth of the Gnomon, which was required.



De-

Demonſtration.

LEt there be made the ſquares N *f*: and G K, and H M, it is evident *(by the 5 of the 2 of Euclid)* that the right angled figure made of G B, B C, and the ſquare N F, is equal to the ſquare made of the halfe line C G, that is the ſquare G K: But the ſquare of B E being equall to the halfe right angle figure B D, it followeth that the ſquare H M is greater then the ſaid ſquare B E by the ſquare N *f*, *(by the 47 of the 1 of Euclid.)* Now then the Gnomon H N M is the halfe of the ſame right angle figure given B D it followeth that the Gnomon G *v* K remaining, is alſo equall to the halfe of the ſame figure given B D, but the ſide I G is equall to the ſide F C by conſtruction; and *v* M to *f* H the half of *q* H, in ſuch ſort that the right angled figures G *v* and *v* K are equall to the right angle Figure H P, to which is added the ſquare P C equall to I *v* coms' R C equall to the Gnomon G *v* K containing as much as the halfe of the right angle figure given B D.

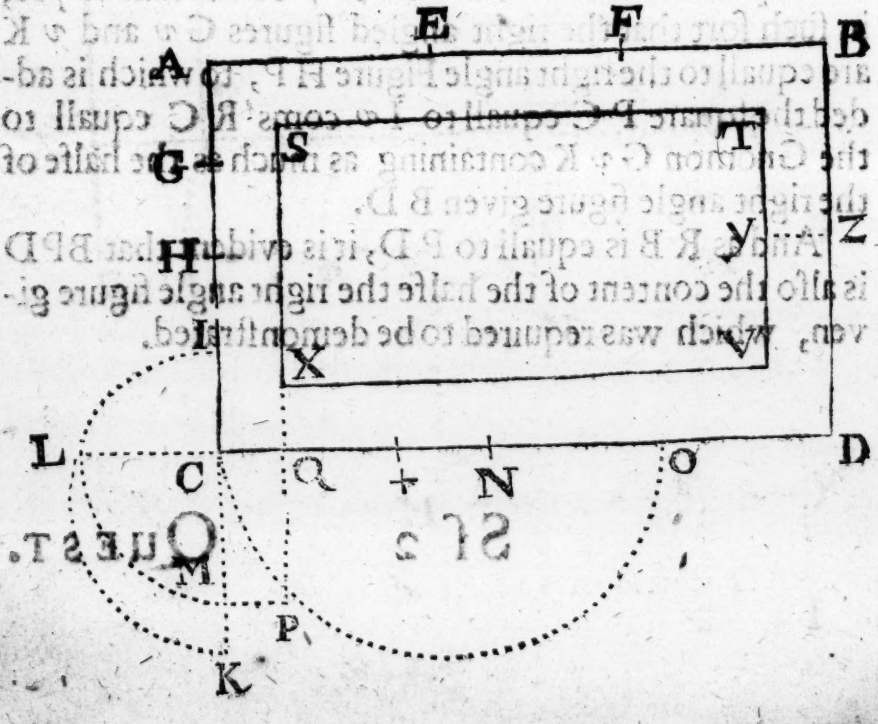
And as R B is equall to P D, it is evident that B P D is alſo the content of the halfe the right angle figure given, which was required to be demonſtrated.

Quest. 99.

How to divide a long Square or right angled Figure into two or more equall or unequall parts, in such sort, as that the part so cut off shall be as it were a band or girdle invironing the rest of the same, and running parallel to all the sides.

The Quadrangle is A B C D, *it is required to divide the same into two unequall parts, in such wise that the part cut off shall be one third part, and the rest shall be just two third parts of the whole content of the Superficies* A B C D.

The Question is, *How much or how long shall be the line* Y Z, *or the breadth of the Band?*



First

FIrſt divide the length into ſuch parts as the whole ſuperficies is required to be divided into, as in this into third parts, as here A E, of A B, divide alſo the breadth A C into 4 equal parts, whereof C I is one fourth part of the breadth, then produce or prolong A C to K, ſo that C K be equall to A E, and produce alſo D C to L, and between I C ¼ of the breadth, and C K one third part of the length: ſeeke a mean proportionall which is C L, then divide the length C D in two equal parts in the point N, and to the halfe line C N add the mean proportionall C L, which coms to O, then divide C O in two equall parts, the midſt is +, and upon + as a centre make a halfe Circle C P O, then from the point M draw a line parallel to the ſide C D, cutting the halfe circle in the point P, then I ſay that the diſtance between the points M & P is the length of the line Z Y, and a line drawn from P to Q parallel to C K, as the line P Q, then is M P or C Q the breadth of the band: then draw the lines parallel to the ſides A B, B C, C D, and A C, as the lines S T, T V, V X, X S, and ſo is the band made about the reſt as was required.

Demonſtration.

IT is evident that the right angled Figure which is made of A E and A C, is equal to the one third part of the whole Superficies A B C D, and the mean proportionall C L, between C I one fourth part of the breadth, and C K one third of the length with A C the breadth doth make a right figure equall to one fourth of the whole, which line C L is alſo equall to the mean proportionall P Q between C Q and Q O, but Q O is one fourth of the whole Circumference or bound, which doth contain one third of the Figure, as alſo the ſquare

of P Q, wherefore Q D multiplyed by Q C doth contain one fourth part of the ſaid third part, wherefore it followeth Q C is the breadth of the ſaid band.

QUEST. 100.

There is a halfe Circle as A B C, *whoſe Diameter is* A B, *upon which is made another leſſer halfe Circle whoſe Diameter is* E A, *ſo that the greater halfe Circle being divided into two equall parts with the line* D C, *doth paſſe through the leſſer Circle in the point* F, *ſo that the part* C F *is 6, and the part* B E *is 9.*

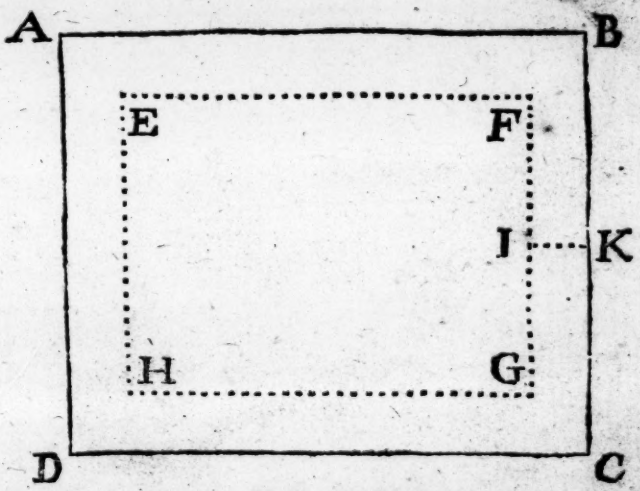
The Queſtion is; *How much is the Diameter of each of theſe two Circles?*

PUt for M D 1(1), then is B D or A D 9 + 1(1), Multiply E D 1(1) by A D 1(1) + 9 the broduct is, 1(2) + 9(1), whoſe ſquare root is √ 1(2) + 9(1), for the mean proportional of the leſſer Circle D F, to this adde C F 6, coms 6 + √ 1(2) + 9(1) for the half Diameter C D or A D, and that is equal to 1(1) + 9, & then is 1(1) equall to 3 for D E, which added to 9 for B E, coms 12 for B D, and 24 for A B, then adde D E 3 to A D 12, coms 15 for A E, which was required.

Thus

Thus I have finiſhed my 100 Queſtions, and although ſome of them do ſeeme to be but of ſmall or no uſe, yet the ingenious ſhal finde in the ſimpleſt of them (one time or other) ſome uſe worth the marking, and to conclude, I have put one Queſtion which I leave unanſwered, only for the practiſe of thoſe who deſire to recreate themſelves in ſuch like novelties.

The Queſtion is thus.



A Gentleman hath a right angled piece of Land as A B C D, *in length 640 foot, and in breadth 480 foot, this piece of Land he would convert into an Orchard and a Garden, but finding the ſame in the Winter to be anoyed with water, he would have the ſame raiſed one foot over the whole ſuperficies. Now it is reſolved to make a Ditch round about the ſame which ſhall be 6 foot deepe, ſo that the Ditch may remain of equall breadth and equall depth, till there be ſo much earth come out of the ſame as will raiſe the whole ſuperficies remaining within the Ditch one foot higher.*

The Queſtion is; *How broad ſhall this Ditch be?*

FINIS.